Week 11-Lab 2: Worksheet 15: Section 4.4 and 4.6

I said: "The worksheet is only a a small sample for Curve sketching. Use the lecture notes, videos and practice more. Knowing the curves help you with mathematical modeling. An immediate application will be Section 4.7, Optimization."

Curve Sketching

The calculus tools we have developed so far enable us to sketch graphs with high accuracy.

Key features of the graph of $f(x)$:

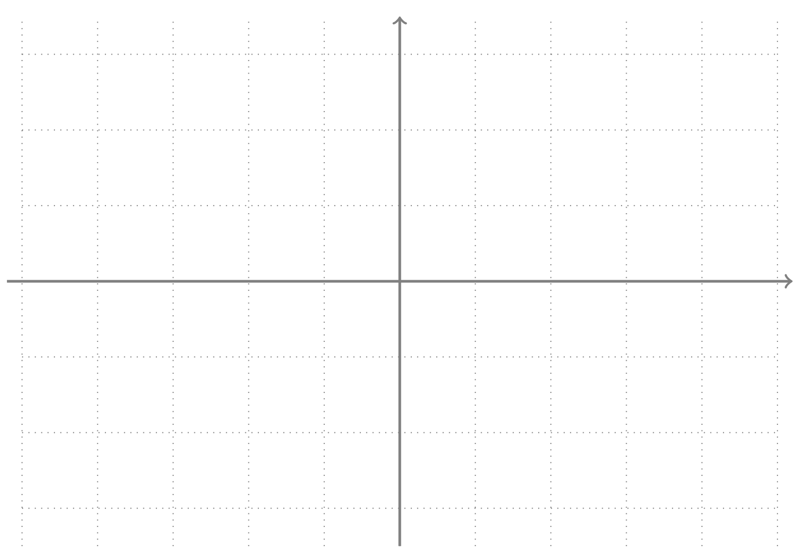
(1) The domain of f — where is f undefined?
(2) Symmetry — is f odd, even, or (usually) neither?
(3) Intervals on which f is increasing or decreasing — use $f'(x)$
(4) Intervals on which f is concave up or down — use $f''(x)$
(5) Local extreme points — use First or Second Derivative Test
(6) Inflection points — points where concavity changes
(7) Horizontal and/or vertical asymptotes

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. Sketch the graph of the function $y = f(x)$ where we know the following:
   (a) $\lim_{x\to-\infty} f(x) = -1$
   (b) $\lim_{x\to\infty} f(x) = 1$
   (c) $f'(-1) = f'(2) = 0$
   (d) $f'(x) < 0$ on intervals $(-\infty, -1)$ and $(2, \infty)$.
   (e) $f'(x) > 0$ on interval $(-1, 2)$.
   (f) $f''(x) < 0$ on intervals $(-\infty, -2.5)$ and $(0.5, 3.5)$
   (g) $f''(x) > 0$ on intervals $(-2.5, 0.5)$ and $(3.5, \infty)$
   (h) $(-2.5, -2)$, $(-1, -3)$, $(0.5, 0)$, $(2, 3)$ and $(3.5, 2)$ are on the graph.
   (Hints: Use a table for f' to find interval of increasing/ decreasing. Use another table for f'', to find concavity.)



2. Let $f(x) = \dfrac{5x}{x^2+3}$. To speed up your calculations, the derivatives are provided:

$$f'(x) = \frac{5(3-x^2)}{(x^2+3)^2} \qquad\qquad f''(x) = \frac{10x(x^2-9)}{(x^2+3)^3}$$

(a) What is the domain of f?

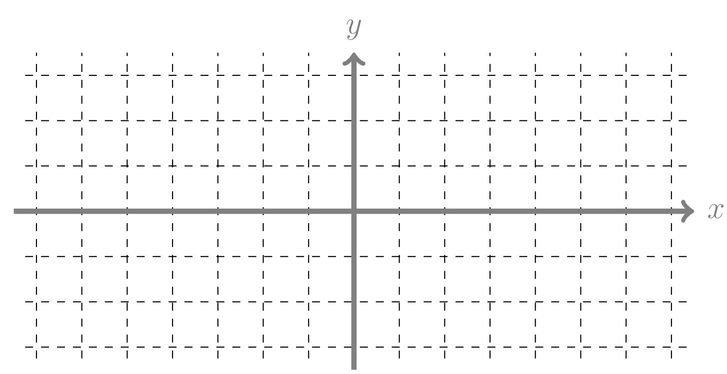
(b) What are the vertical asymptotes and horizontal asymptotes of f?

(c) On what intervals is f increasing? decreasing?

(d) On what intervals is f concave up? concave down?

(e) At what point(s) does f have a local maximum? local minimum? inflection?

(f) Sketch the graph of f noting the above information:



GroupWork Rubrics:

Preparedness: ____/0.5, Contribution: ____/0.5, Correct Answers: ____/0.5

Individual Portion of the Worksheet

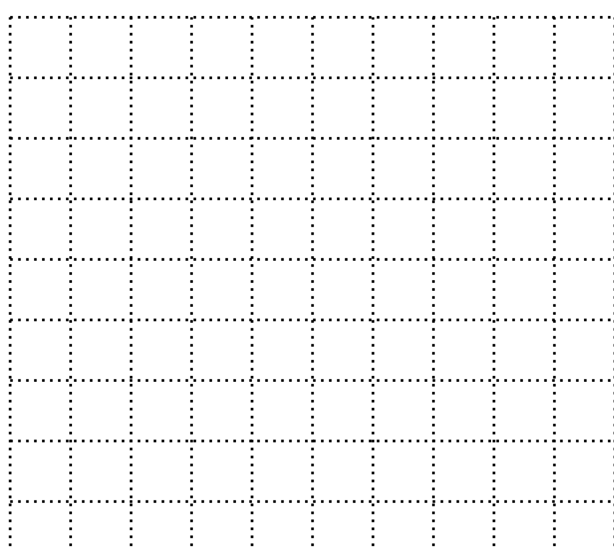
Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2^{nd} lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

Background Story: Use the information on the Worksheet 14 Problem 5 Part A and B to complete Problem 3.

Questions: Let $f(x) = x^{\frac{7}{3}} - 7x^{\frac{4}{3}}$.

3. *(0.75 points)* Sketch the graph of $f(x)$ noting the intervals of increasing/ decreasing, concavity and extremum of the function.



4. This problem is about the function $f(x) = \dfrac{(1-x)^3}{x^2}$. To assist your calculations, the factored derivatives are provided:

$$f'(x) = \frac{-(1-x)^2(x+2)}{x^3} \qquad f''(x) = \frac{6(1-x)}{x^4}$$

(A) *(0.25 points)* What is the domain of f?

(B) *(0.5 points)* What are the vertical and horizontal asymptotes of f?

(C) *(0.5 points)* On what intervals is f increasing? decreasing?

(D) *(0.25 points)* On what intervals is f concave up? concave down?

(E) *(0.25 points)*At what point(s), if any, does f have a local maximum?

(F) *(0.25 points)* At what point(s), if any, does f have a local minimum?

(G) *(0.25 points)* What are the inflection points of f, if any?

(H) *(0.5 points)* Use the information in parts (a)–(g) to sketch the graph of f on the axes provided below.