Week 13-Lab 1: Worksheet 17: Section 4.7

They asked: "All of the week's individual questions are given in this worksheet?!" I said: " Yes! The optimization word problems are really important and one of the few real-life application of Calculus I that we can show you. We are putting a lot of emphasis on them. You can continue some of the problems on the next lab day but try them on your own as well. The next worksheet is just a groupwork."

Solving Optimization Problems

(1) Draw a diagram (if applicable) and fix notation.

(2) What are you attempting to optimize? (the "objective function")
What are the constraints?

(3) What are the constants and variables?
What are the domains for the variables?

(4) Use the constraints to rewrite the objective function in terms of a single variable.

(5) Use calculus to find the absolute minimum or maximum of the objective function on the appropriate domain.

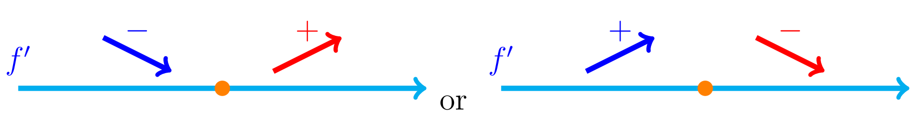
What are the best practices in finding the characterization of critical numbers:

(A) If the domain is a closed interval, using closed interval method. (Checking the end points and critical numbers.)

(B) If the domain is a bounded open interval, then see if you can add the end points, then use (A).

(C) If the domain is an open interval, bounded or unbounded, that can not be extended or restricted to a closed interval, then check the number of local extrema. If only one local extrema exists, then that local extrema is an absolute extrema. This process can be done by first derivative or second derivative test.

First derivative test:



(D) There are yet other situations that is solved by looking at the graph of the function.

Group Work Portion of the Worksheet

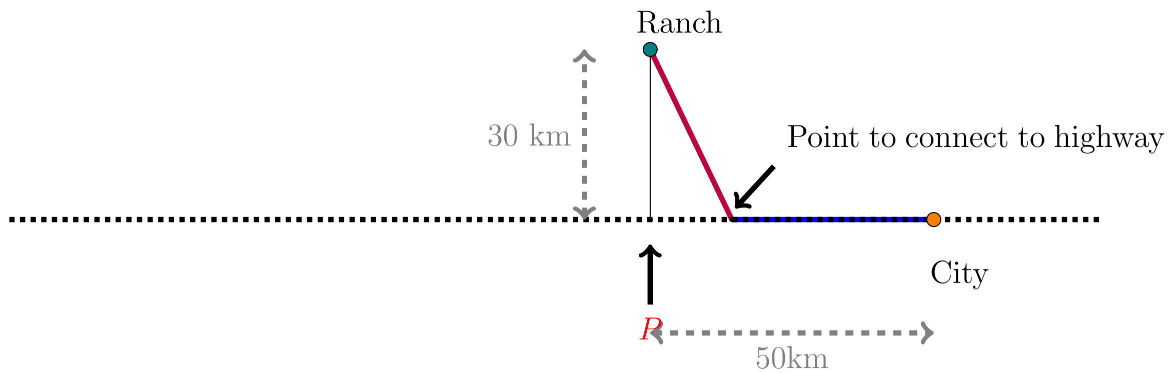
Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** Watching this video can help: https://mediahub.ku.edu/media/t/0_oj45rrk9.

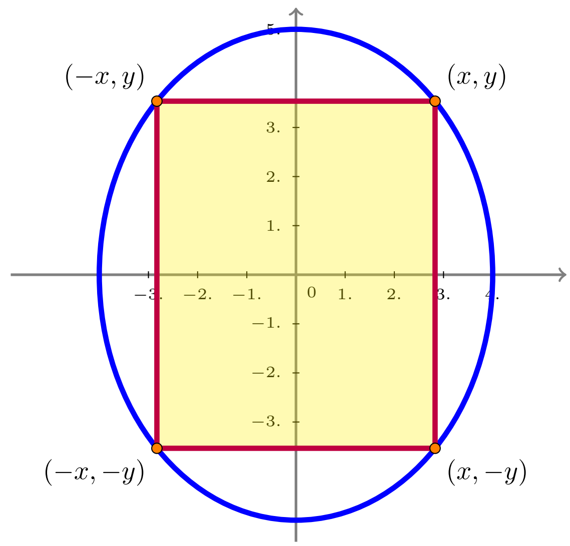
   Questions:

   A straight road is to be built connecting a ranch to a highway which leads to a city. The speed limit on the road is to be $50\,km/hr$ and the speed limit on the highway is $130\,km/hr$. The ranch is $30\,km$ from the point P on the highway which is the closest point on the highway to the ranch and is $50\,km$ from the city. At what point on the highway should the straight road be built to which allows for the quickest drive from the ranch to the city?



2. **Background Story:** Sometimes the least complicated way is using trigonometry and parametrization if you know those from precalculus. Also can we assume area zero is possible and have a closed interval for domain?

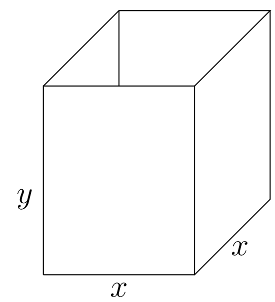
   Questions: Find the area and the dimensions of the largest rectangle that can be inscribed in the ellipse



$$\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$$

Watch the change in area:
https://ggbm.at/nmu4b5zb
https://mediahub.ku.edu/media/t/1_z1nhfp5m

3. An open box in the shape of a rectangular prism with a square base is to be constructed so its surface area is 400 square inches. What dimensions will maximize its volume? And what is the maximum volume?



Video:https://youtu.be/Le0PrwmlmL8

GroupWork Rubrics:

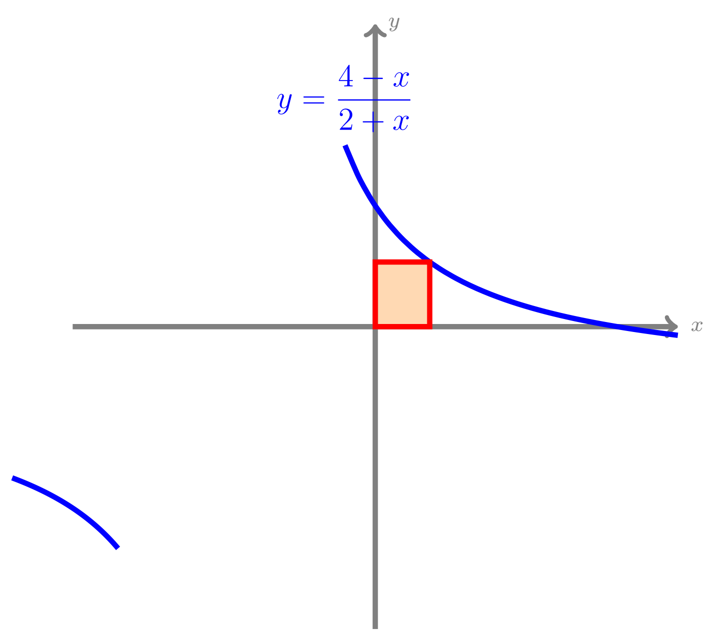
Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of the Worksheet

Name: ________________

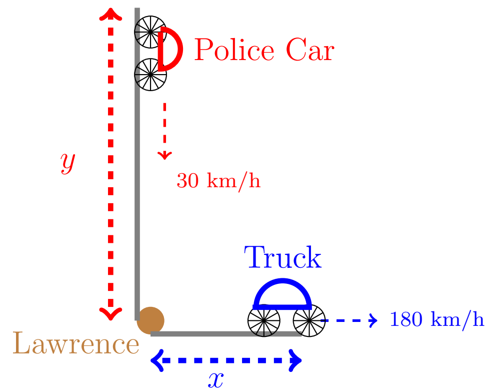
Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

4. *(3.5 points)* Find the maximum area of a rectangle which can be inscribed in the region bounded by $y = \dfrac{4-x}{2+x}$, the x-axis, and the y-axis in the first quadrant.



Watch how area changes using this applet: https://ggbm.at/jpq63tf4.
https://mediahub.ku.edu/media/t/1_4aulw2qd.

5. *(3.5 points)* A police car starts traveling south, from a position 13 km north of Lawrence, toward Lawrence at the constant speed of 0.5 km/min pursues a truck that is traveling east, from a position 2 km east of Lawrence, away from Lawrence at the constant speed 3 km/min (See figure to the right). When is the distance between the two vehicles minimum?



Watch the chase here: https://ggbm.at/kfwvhwvq

The police car problem is good both for optimization and related rate: https://youtu.be/4C4d8S9drt8