Week 13-Lab 2: Worksheet 18: Section 4.8 and 5.1:

I said: " Integration is arguably the most fundamental concept in Calculus. It will show up in every course where Calculus is a prerequisite. Please keep going to classes and completing your assignments. For us, it has been a year with post lock-down deficiencies and extreme reductions in resources. Despite it all, watching your progress and the huge improvement in your math knowledge and skills keep us going. Please continue the hard work. Don't slow down! "

Velocity and Distance

The area under the graph of $v(t)$ on a time interval $[a, b]$ measures the net distance traveled, or displacement, between times a and b.

$$\begin{aligned}\text{Units of area under the graph of } v(t) &= \text{units of } t \times \text{units of } v(t)\\ &= \text{time } \times \frac{\text{distance}}{\text{time}}\\ &= \text{distance.}\end{aligned}$$

Summation Notation:

The notation $\sum_{j=m}^{n} a_j$ means $a_m + a_{m+1} + a_{m+2} + \ldots + a_{n-1} + a_n$.

- $\sum$ is the Greek letter Sigma (mnemonic for "sum.")
- The notation $\sum_{j=m}^{n}$ tells us to start at $j = m$ and to end at $j = n$.
- a_j is called the **general term** and j is the **summation index**.

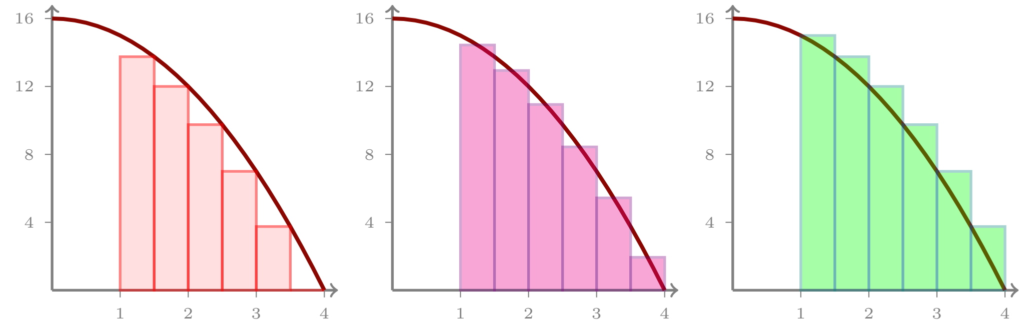
Riemann Sum:

The estimate for the area under the graph of a continuous, positive function $f(x)$ on an interval $[a, b]$ is

$$\sum_{i=1}^{n} f(x_i)\,\Delta\,x = f(x_1)\,\Delta x + f(x_2)\,\Delta x + \ldots + f(x_n)\,\Delta x$$

and the exact area is

$$\begin{aligned}A &= \lim_{n\to\infty} R_n = \lim_{n\to\infty} \left(f(x_1)\,\Delta x + \ldots + f(x_n)\,\Delta x\right)\\ &= \lim_{n\to\infty} \sum_{j=1}^{n} f(x_j)\,\Delta x.\end{aligned}$$



Right-Hand point approximation: R_n Mid point approximation: M_n Left-Hand point approximation: L_n

Approximation of the net area over $[a, b]$ using Riemann sums.

- If n is the number of divisions, the length of sub-intervals is $\Delta x = \dfrac{b-a}{n}$.
- Choose sample point for each sub-interval. The height of each rectangle over the ith sub-interval is $f(x_i)$ and the area is $f(x_i)\Delta x$.
- Note that if you are computing R_n, L_n and M_n (right-end point, left-end point and mid-point approximation with n intervals), then for

  L_n: $x_0 = a,\ x_1 = a + \Delta x,\ \cdots\ x_i = x_0 + i\Delta x\ \cdots\ x_n = x_0 + n\Delta x$

  R_n: $x_0 = a + \Delta x,\ x_1 = a + 2\Delta x,\ \cdots,\ x_i = x_0 + (i+1)\Delta x,\ \cdots,\ x_n = b$

  M_n: $x_0 = a + \frac{\Delta x}{2},\ x_1 = a + \frac{\Delta x}{2} + \Delta x,\ \cdots,\ x_i = x_0 + \frac{\Delta x}{2} + i\Delta x,\ \cdots,\ x_n = x_0 + \frac{\Delta x}{2} + n\Delta x.$
- Sum all the areas.

Newton's Method (Section 4.8)

To approximate a root of $f(x)$, choose an initial value x_0. Generate successive approximations of the root through the equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Sometimes, Newton's Method does not approximate a root.

GroupWork Rubrics day 2:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** Two points to remember: ① Remember the distance traveled in small interval of time, Δs_i, can be approximated by $\Delta s_i \approx v(t_i)\Delta t$. ② The sum of all of these small lengths approximate the total displacement and is a Riemann Sum.

   Questions: A car starts moving and continues with positive acceleration for two seconds. The **velocity** of a car is recorded at half-intervals(in meters per second). Use the left- and the right-endpoint approximation to estimate a lower and an upper bound for the net **distance** traveled during the first 2 seconds.

t(in seconds)	0	0.5	1	1.5	2
$v(t)$(in meters per seconds)	0	2	6	11	20

2. **Background Story:** ① The Riemann sum approximations, for a continuous function $f : [a, b] \to \mathbb{R}$, are categorized by the number of sub-intervals n and the sample points. Some examples are L_n which is the the approximation using n intervals and the **left end point** of each interval as samples points. R_n and M_n are similarly defined for approximations using **right-hand** points and **mid-points**.

   ② The net area $=$ (area under the graph of f and above x-axis between $x = a$ and $x = b$) $-$ (area above the graph of f and below the x-axis).

   Questions: For $f(x) = x^2 - 7x + 6$ on the interval $[1, 7]$. Calculate R_4, L_4, and M_4. Sketch the graph of f and the rectangles that make up each approximation.

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3. Understand $\sum$ notation .

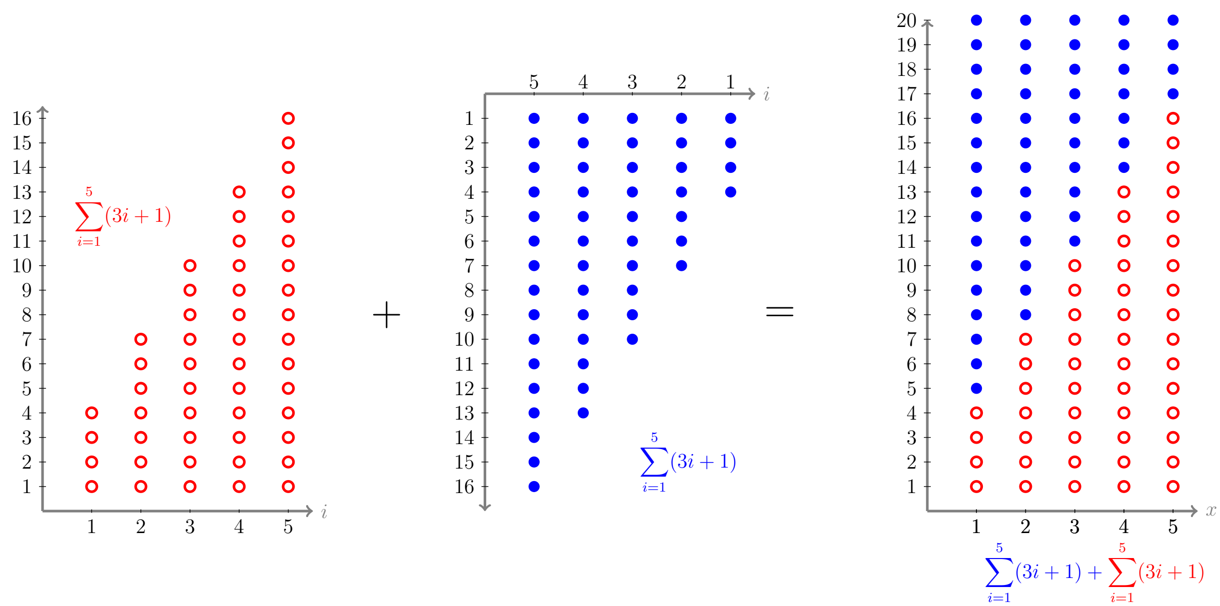
(A) Expand $\displaystyle\sum_{n=1}^{5}(3n+1)$ and evaluate it.

(B) Expand $\displaystyle\sum_{n=0}^{5}(3n+1)$ and evaluate it.

(C) *(Calc 2)* Find the values of c and d such that $\displaystyle\sum_{n=1}^{5}(3n-2)=\sum_{n=c}^{d}(3n+1)$.

(D) *(Computer science)* Find a formula for $\displaystyle\sum_{i=1}^{n}(3i+1)+\sum_{i=1}^{n}(3i+1)$ using the following pattern.

Then find a formula for $\displaystyle\sum_{i=1}^{n}(3i+1)$.



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4. Let $f(x) = x^2 - 8$. Compute the root of this polynomial using Newton's method.

Note that the Newton's formula for the polynomial is $x_{n+1} = x_n - \dfrac{x_n^2 - 8}{2x_n}$

(A) Start point: $x_0 = 1$.

$$x_1 = x_0 - \frac{x_0^2 - 8}{2x_0} = 1 - \frac{1^2 - 8}{2(1)} = 4.5$$

$$x_2 = x_1 - \frac{x_1^2 - 8}{2x_1} = 4.5 - \frac{4.5^2 - 8}{2(4.5)} = 3.1389$$

$x_3 =$

$x_4 =$

$x_5 =$

(B) Use your calculator to find the roots of $x^2 - 8 = 0$. What is the accuracy of the solution in (A) after 5 iterations?

(C) Start point: $x_0 = -1$.

$x_1 =$
$x_2 =$
$x_3 =$
$x_4 =$
$x_5 =$

(D) Is the answer in (C) the same as in (A)?

(E) Start point: $x_0 = 0.5$.

$x_1 =$
$x_2 =$
$x_3 =$
$x_4 =$
$x_5 =$