Week 2-Lab 1: Worksheet 2: Precalculus Review

They asked:"How do I solve problems that I never encountered before?" I answered: "Learn the related material from lecture and lab; read the problem and identify the objective; then identify intermediate objectives and use all your math knowledge, from the beginning of your time to now, logic and common sense to translate each intermediate objective to math and to solve. Remember solving a problem similar to a problem that you have seen before is not problem solving. Problem solving is the ability to read and parse important information, pinpoint what you are being asked and connect that to tools that you have learned your entire life. "

Short Descriptions and Formulas

Laws of Exponents

Let a and b be positive numbers and let x and y be real numbers. Then,

1. $b^x \cdot b^y = b^{x+y}$
2. $\dfrac{b^x}{b^y} = b^{x-y}$
3. $(b^x)^y = b^{xy}$
4. $b^0 = 1$
5. $b^1 = b$
6. $(ab)^x = a^x b^x$
7. $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

Laws of Logarithms

If m and n are positive numbers and $b > 0$, $b \neq 1$, then

1. $\log_b(mn) = \log_b(m) + \log_b(n)$ Log of a product is the sum of logs
2. $\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n)$ log of a quotient is difference of logs
3. $\log_b\left(m^n\right) = n\log_b(m)$ log of an exponential is the product of exponent $\times$ log of base of the exponential
4. $\log_b(1) = 0$ log of one is zero
5. $\log_b(b) = 1$ log of the base is one
6. $\log_c(x) = \dfrac{\log_b(x)}{\log_b(c)}$ Change of base formula

Fun Fact:

Numbers 1-6 of laws of logarithm are consequences of laws of exponents.

- **Show "Law of exponent 1 $\implies$ Law of Logarithm 1":**
- To **combine** two or more logs, make sure that the coefficients become exponents inside the log, then use the log of sum and difference law.
- To **expand** a log, use the law of sum and differences first, and then use the law of exponents and simplify using inverse function property.

Constant Velocity:

- If the velocity of a particle is constant then its average velocity and instantaneous velocity is the same. In that case, $v = \dfrac{s}{t}$, where v is the constant velocity, s is the position from origin, and t is time.
- How else can you write $v = \dfrac{s}{t}$? Hint: Solve for s and t.

Group Work Portion of Worksheet

Names: _____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** When two variables are related their rates are related too. To find the relation between rates, the first step is finding the relationships between variables themselves. The following problem promotes that. Note that in this problem, we are asking you to solve for a variable in terms of the other one. The variable that you solve for is the dependent variable and the other variable is the independent variable.

   Questions: Albert lets go of his balloon and the balloon starts rising vertically at a **constant speed** of 15 feet per second. Mom who is standing 60 feet from Albert is looking directly at the balloon, which was originally in her horizontal line of sight, stays frozen in her place and keeps looking straight at the balloon as it is rising. Let y be the vertical distance between Albert and the balloon, in feet, and θ be the angle of mom's line of sight with horizon.

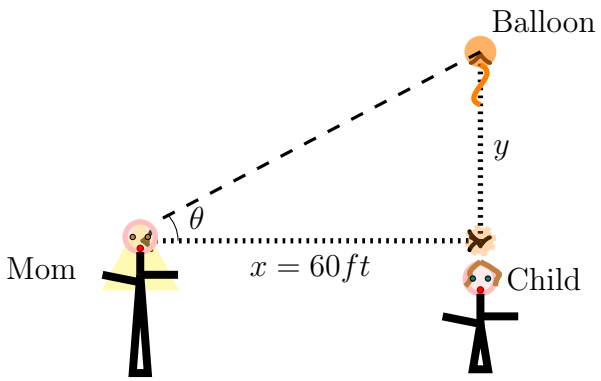
   (a) Express the distance between the mom and the balloon, D, as a function of y.

   (b) Express y as a function of t, time in seconds.

   (c) Express D as a function of t.

   (d) Express θ as a function of y.

   (e) Express θ as a function of t.

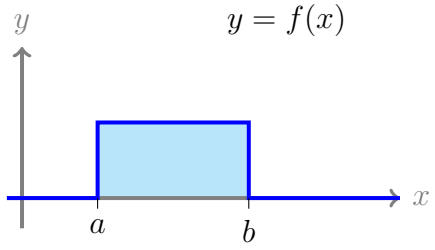


https://mediahub.ku.edu/media/t/0_3r1qegbd

2. **Background Story:** The probability distributions are used in many STEM fields. Most of you are familiar with normal distribution.[1] In earlier physics and engineering courses, you will see uniform probability distribution. A function $f(x) = \begin{cases} k & \text{When } a \le x \le b \\ 0 & \text{Otherwise} \end{cases}$ is a **uniform probability distribution function** if the **area** entrapped between the graph of the function, x-axis, $x = a$ and $x = b$ is **one**. The following problem will build skills that you need to understand the distribution and also enforces the concept of functions.

Questions: Consider $f(x) = \begin{cases} k & \text{When } a \le x \le b \\ 0 & \text{Otherwise} \end{cases}$.

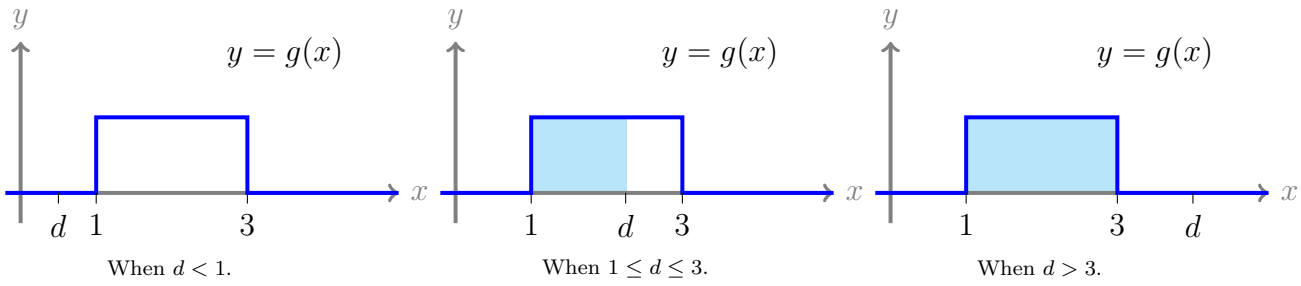
(A) What is the area entrapped between the graph of the function, x-axis, $x = a$ and $x = b$ in terms of a, b and k? (The shaded area in the picture.)



(B) For what value of k is $f(x)$ a probability distribution function?

(Give a value in terms of a and b.)

(C) Consider $g(x) = \begin{cases} 0.5 & \text{When } 1 \le x \le 3 \\ 0 & \text{Otherwise} \end{cases}$. Define function $A(d)$ to be the area entrapped between the graph of the function g and x-axis when $x < d$ in term of d. What is $A(d)$ if $1 \le d \le 3$? What is $A(d)$ if $d < 1$? What is $A(d)$ if $d > 3$?



(D) Replace d with x in your answers for Part (C) and write $A(x)$ as a piecewise function.

[1] We discuss a few fact about normal distribution in each of the calculus sequence. https://youtu.be/6rEcY099hIU

3. **Background Story:** There are instances that we need to find the compositions of two or more piecewise-defined functions. In Computer science we hear all about zeros and ones. What are zeros and ones in a computer? They are voltages or currents. But does that mean a voltage has to be zero to render a zero in binary system? It turns out that a range of voltage is accepted as zero and a range of voltage is accepted as one. The next problem discusses this ranges on an alternating voltage. Note that this type of problems is not restricted to electrical or computer engineering.

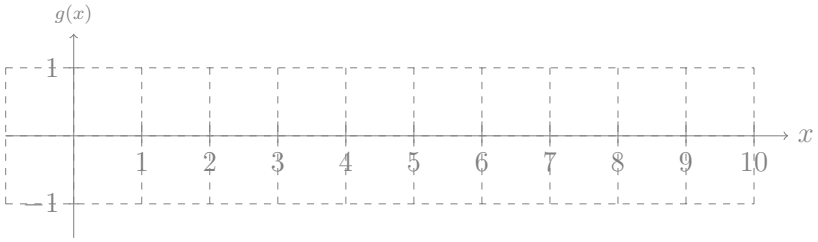
Questions: Consider $g(x) = \cos(\frac{\pi x}{4})$ restricted to the domain $[0, 8]$.

(A) For what values of x, $g(x) = \dfrac{1}{2}$?

(B) For what values of x is $g(x) > \dfrac{1}{2}$?

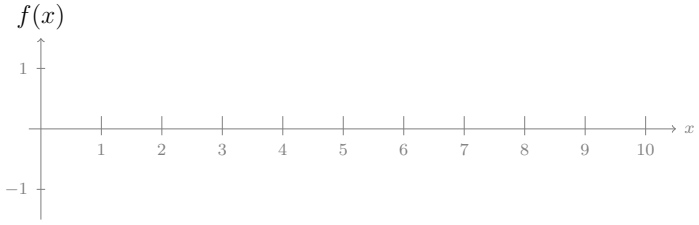
(C) For what values of x is $g(x) \leq \dfrac{1}{2}$?

(D) Graph $g(x)$ and mark the part(s) where $y > \dfrac{1}{2}$.



(E) Use the graph in Part (D) to sketch the graph of $f(x) = \begin{cases} 1 & \text{when } \cos\left(\frac{\pi x}{4}\right) > \dfrac{1}{2} \\ 0 & \text{when } \cos\left(\frac{\pi x}{4}\right) \leq \dfrac{1}{2} \end{cases}$

restricted to the domain $[0, 8]$.



(F) *(0.5 points)* Fill in the blank with interval(s): $f(x) = \begin{cases} 1 & \text{when } x \text{ is in } \underline{\quad\quad} \\ 0 & \text{when } x \text{ is in } \underline{\quad\quad} \end{cases}$

(G) Consider the function $h(x) = \begin{cases} 1 & \text{when } x > 1/2 \\ 0 & \text{when } x \leq 1/2 \end{cases}$. How are f, g and h related?

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of Worksheet

Name: _____

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

4. **Background Story:** One of the important subjects in Calculus I is optimization which has applications in STEM and business. To be able to solve optimization problems in Calculus I, we will need to simplify expressions and solve equations.

   Questions: Consider the function

   $$f(x) = 4(x+1)^{2/3} + \frac{28x}{(x+1)^{1/3}}.$$

   (A) *(2 points)* Simplify and factor the function f as much as possible in form of a quotient function.

   (B) *(0.5 points)* Find the **zeros** of f.

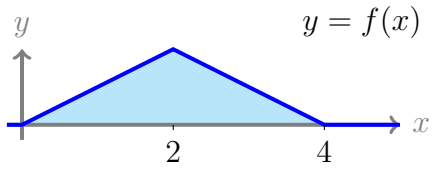
   (C) *(1 point)* Find the **domain** of of the function.

Video: https://mediahub.ku.edu/media/t/0_40cgmxuc

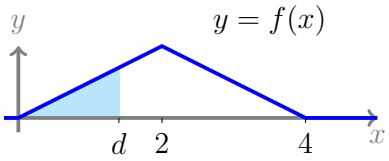
5. **Background Story:** Computing the area under the graph and above the x-axis for $a \leq x \leq x$ and the area below the x-axis and above the graph for $b \leq t \leq x$ is really important in multiple field of STEM. Let do another example of it.

Questions: Consider $f(x) = \begin{cases} 0.5x & \text{When } 0 \leq x \leq 2 \\ -0.5(x-4) & \text{When } 2 \leq x \leq 4 \\ 0 & \text{Otherwise} \end{cases}$.

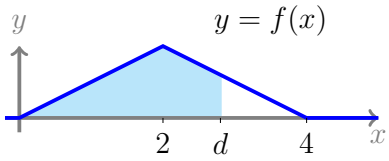
(A) *(0.5 points)* What is the area entrapped between the graph of the function, x-axis, $x = 0$ and $x = 4$? (The shaded area in the picture.)



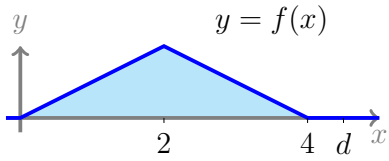
(B) *(2.25 points)* Consider $f(x) = \begin{cases} 0.5x & \text{When } 0 \leq x \leq 2 \\ -0.5(x-4) & \text{When } 2 \leq x \leq 4 \\ 0 & \text{Otherwise} \end{cases}$. Define $A(d)$ to be the area entrapped between the graph of the function g and above the x-axis when $0 < x < d$ in term of d. What is $A(d)$ if $0 \leq d \leq 2$? What is $A(d)$ if $2 < d < 4$? What is $A(d)$ if $d > 4$?



When $0 \leq d \leq 2$.



When $2 < d < 4$.



When $4 < d$.

(C) *(0.75 points)* Replace d with x in your answers for Part (C) and write $A(x)$ as a piecewise function.

Math 125 and material, open letter to my students

Dear friends of Math 125,

We always receive reports of different websites that take our material and sell them for profit to students. They also pay small fees or allow free access to our students to obtain these material. Please note that this activity is illegal and is a form of intellectual property theft of your instructors. Please don't be tricked by these websites. We spend long hours outside our working hours to create these material so our students can use them for **free**.

We also receive reports of tutors on social media platforms such as Groupme who use our previous semester material and, instead of really helping you, they just give you the solution to material. This activity is academic dishonesty. Some of these tutors are paid by small groups and they seem to help everyone in the social media. Please only get help from those tutors if they are willing to be on the same social media platform(s) as your instructors or add us to the social media platform they are using. Please don't get tricked by their gimmicks to create a divide between you and your instructors. That "divide" is how they profit with very small amount of work. We are all about helping each other out in a way that helps you learn more. So please have us involved in your problem solving process.