Week 15-Lab 2: Worksheet 21: Sections 5.4, 5.5 and 5.7

We said: "Hang in there! Complete the course strong!"

The Fundamental Theorem of Calculus I (FTC-1):

If f is continuous on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is **any** antiderivative of f, that is, $F' = f$.

FTC-1: Alternative Formulations:

We frequently use the notation $F(x)\Big|_a^b$ to stand for $F(b) - F(a)$, so that FTC-1 can be rewritten as

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b$$

FTC-1 says that definite and indefinite integrals are related as follows:

$$\int_a^b f(x)\,dx = \int f(x)\,dx\,\Big|_a^b$$

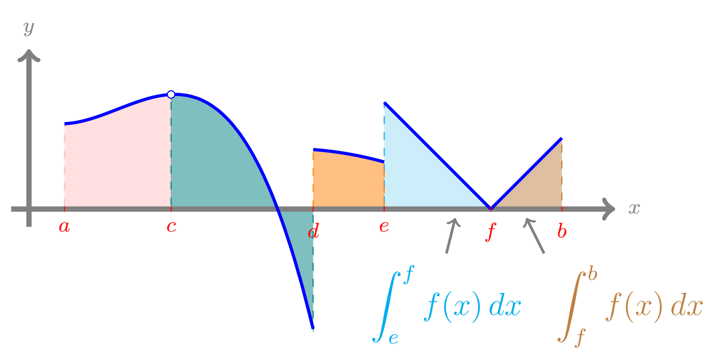
Here is another way of writing FTC-1:

$$\int_a^b F'(x)\,dx \;=\; F(x)\Big|_a^b \;=\; F(b) - F(a).$$

This version of FTC is often referred to as the **Net Change Theorem**, because it says that the integral of $F'(x)$ - that is, the integral of the rate of change of $F(x)$ -is the net change in F.

Finite Numbers of Holes and Jumps:

The statement of FTC-1 applies only to continuous functions, but in fact it can be used to integrate functions whose only discontinuities are a **finite** number of holes or jumps. For example:



$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^d f(x)\,dx + \int_d^e f(x)\,dx + \int_e^f f(x)\,dx + \int_f^b f(x)\,dx$$

and each the five integrals on the right can be evaluated using FTC.

Warning: FTC-1 cannot be used if $f(x)$ has an infinite discontinuity (vertical asymptote). We will explore this further in MATH 126.

Cumulative Area Function:

Graph of $f(t) = \frac{d}{dx} A_f(x)$	**Area function** $A_f(x) = \int_a^x f(t)\,dt$
Above the x-axis	Increasing
Below the x-axis	Decreasing
Zero	Local extremum
Increasing	Concave up
Decreasing	Concave down

The Fundamental Theorem of Calculus II (FTC-2):

Suppose f is continuous on the interval $[a, b]$. Then, for all x in $[a, b]$:

$$\frac{d}{dx}\left(A_f(x)\right) = \frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x).$$

$$\frac{d}{dx}\Big(A_f(g(x)) - A_f(h(x))\Big) = f(g(x))g'(x) - f(h(x))h'(x), \text{ where } f,\ g \text{ and } h \text{ are continuous.}$$

Substitution for Indefinite Integrals:

If $u = g(x)$ and $du = g'(x)\,dx$ then we can rewrite this equation as

$$\int f'(g(x))\,g'(x)\,dx \;=\; \int f'(u)\,du \;=\; f(u) + C \;=\; f(g(x)) + C.$$

The Substitution Method for Definite Integrals

If $g'(x)$ is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. Use definite integrals, antiderivatives, to compute the definite integrals.

   (A) $\displaystyle\int_0^{\pi/2} \cos(t)\,dt$

   (B) $\displaystyle\int_1^4 \sqrt{x}(x-2)\,dx$

   (C) $\displaystyle\int_0^1 \frac{1}{x^2+1}dx$

2. **Background Story:** From Review material!

   Calculate $\dfrac{d}{dx}\left(\displaystyle\int_{-x^2}^{\sqrt{x}} \tan^5(t)\,dt\right)$ on the domain $0 \leq x \leq \sqrt{\dfrac{\pi}{3}}$. Be sure to quote any theorem used in completing the calculation. **video:** https://mediahub.ku.edu/media/t/1_us3hg8nc

3. **Background Story:** From the review again! Make sure to know the relationship between A_f, f and f'. Use first and second derivative of A_f to find the shape.

   Sketch the graph of an increasing function f such that both $f'(x)$ and $A_f(x) = \displaystyle\int_0^x f(t)\,dt$ are decreasing.

4. **Background Story:** From the review again! You did similar question when reversing the derivative, now use u-substitution for definite integrals.

   $\displaystyle\int_0^9 f(x)\,dx = 4$ show that $\displaystyle\int_0^3 xf(x^2)\,dx = 2$

5. **Background Story:** Practice some of the review questions! These are the set of definite and indefinite integrals in the review that need method of substitution. Please for each integral, specify $u =$ _____, $du =$ _____, the new integral bounds if they apply and the transformed integral. The solve.

Questions: Evaluate the following definite and indefinite integrals:

(A) $\displaystyle\int (5x-4)^8\,dx$

(D) $\displaystyle\int x^5\sqrt{x^3+1}\,dx$

(B) $\displaystyle\int \frac{dt}{\sqrt{t-5}}$

(E) $\displaystyle\int \cot(x)\,dx$

(C) $\displaystyle\int x^2\sqrt{x^3+1}\,dx$

(F) $\displaystyle\int \sin^7(x)\cos(x)\,dx$

(G) $\displaystyle\int \sec^2(x)e^{\tan(x)}\,dx$

(J) $\displaystyle\int_1^e \frac{\ln(x)}{x}\,dx$

(H) $\displaystyle\int_4^9 (x-8)^{-\frac{2}{3}}\,dx$

(K) $\displaystyle\int_0^{\frac{\pi}{2}} \cos^3(x)\sin(x)\,dx$

(I) $\displaystyle\int_9^{15} (x-8)^{-\frac{2}{3}}\,dx$

GroupWork Rubrics:

Preparedness: —–/0.5, Contribution: —–/0.5, Correct Answers: —–/0.5

Individual Portion of the Worksheet

Name: ________________

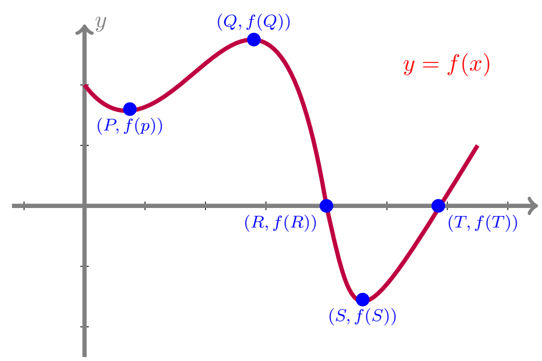
Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: —–/0.5, Contribution: —–/0.5, Correct Answers: —–/0.5

6. **Background Story:** By FTC 2, $f(x) = \frac{d}{dx}A_f(x)$.

   Questions: Let $A_f(x) = \int_0^x f(t)\,dt$ where $y = f(x)$ is graphed below.



   (A) *(0.75 points)* Does A_f have a local minimum at S? Explain your answer.

   (B) *(0.75 points)* Where does A_f have a local minimum? Explain your answer.

   (C) *(0.75 points)* Where does A_f have a local maximum? Explain your answer.

   (D) *(0.75 points)* True or False? $A_f(x) < 0$ for all x in the interval shown. Explain your answer.

7. Evaluate the following integrals using u-substitution.

(a) *(2 points)* $\displaystyle\int_{-\frac{3}{\sqrt{7}}}^{0} x(1-7x^2)^6\,dx$

(b) *(2 points)* $\displaystyle\int_{0}^{\frac{\pi}{2}} \cos(x)\cos(\sin(x))\,dx$