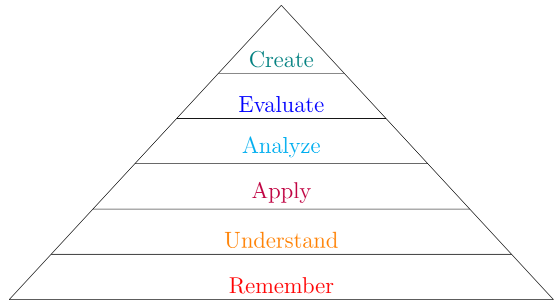
Week 3-Lab 1: Worksheet 3: Sections 2.1 and 2.2

I said: "Look at this Pyramid, what you are doing in any lecture is the two lower steps of the pyramid. When assignments are very similar to lecture problems, you are doing the third step. When assignments are different from lecture but guided (some of the worksheets) you are on the fourth and fifth steps. The top step is only achieved when assignments are different from lecture with little guidance. You need to try all steps to really learn." They asked: " Do you have recommendation for tutors?" I said: " Our help room, our office hours, SIs, SAC and school of engineering tutoring are free. Try those first."



Short Descriptions and Formulas

One-Sided Limit

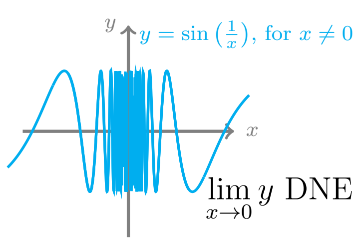
- $\lim\limits_{x\to c^-} f(x) = L$ The left limit of f is L if the y-values converge to L as x approaches c through values **less than** c.
- $\lim\limits_{x\to c^+} f(x) = L$ The right limit of f is L if the y-values converge to L as x approaches c through values **greater than** c.

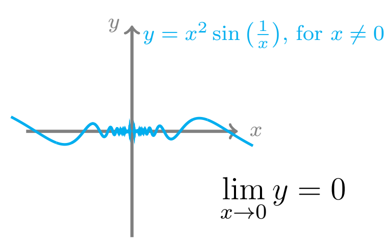
Two-Sided Limit

$\lim\limits_{x\to c} f(x) = L$ The two-sided limit of f is L **ONLY IF** both one-sided limits equal L.

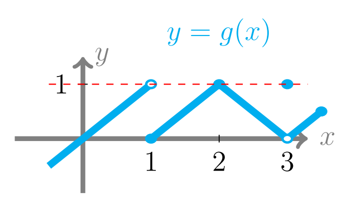
Graphs and Tables

- In familiar cases or when graphs have enough information, you can use graphs or tables to estimate.
- There are situations where getting enough samples for a table may be difficult. In the sections to come, we find methods to find limits without worrying about finding samples.
- Example when limit doesn't exists because of non-decaying infinite oscillations and examples that limit exists despite oscillations because the amplitude is controlled.





- An example when the graph is helpful in finding the limit.



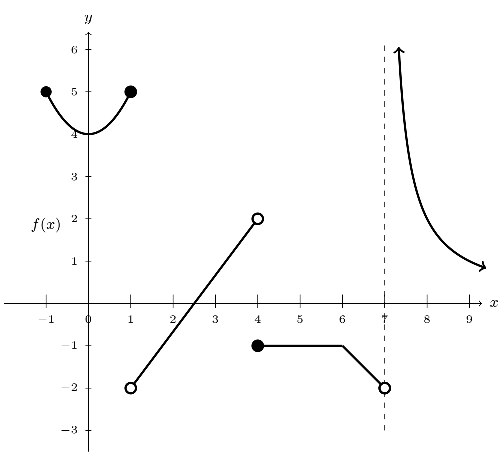
Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** The next problem is about left and right limits. Find when one exists or does not. The rules of thumb: ① If there is a jump in the graph but both right and left limits exist but the limit does not. ② If the function has a vertical asymptote and the output is tending to infinity on the left or right, then either left or right limit or both don't exists. ③ There are other reasons for limit not to exist. Look at the examples in Page 1.

   Questions: The function $y = f(x)$ is graphed below. Evaluate the following expressions or enter DNE.



(i) $f(-1) =$

(ii) $f(0) =$

(iii) $f(1) =$

(iv) $f(7) =$

(v) $\lim_{x\to -1^+} f(x) =$

(vi) $\lim_{x\to 1^-} f(x) =$

(vii) $\lim_{x\to 1^+} f(x) =$

(viii) $\lim_{x\to 4^-} f(x) =$

(ix) $\lim_{x\to 4^+} f(x) =$

(x) $\lim_{x\to 7^-} f(x) =$

2. **Background Story:** Often in industry we are looking at rates of change. In the next problem, we look at average rate of change of a sphere when radius is changing. First a numerical interval and then parametric interval.

   Questions: The volume of a sphere of radius R is $V = \frac{4}{3}\pi R^3$.

   (A) What is the average rate of change of the volume when the radius decreases from $R_1 = 7\ cm$ to $R_2 = 3\ cm$?

   (B) What is the average rate of change of the volume when the radius decreases from $R_1 = 3\ cm$ to $R_2 = 7\ cm$?

   (C) Compute the average rate of change of volume of the sphere when radius is changing from $R_1 = a$ to $R_2 = b \neq a$ and then the identity $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ to simplify the expression.

3. Sketch the graph of a **single function** f satisfying all of the following properties:

(i) $\lim_{x \to -2^-} f(x) = \infty$

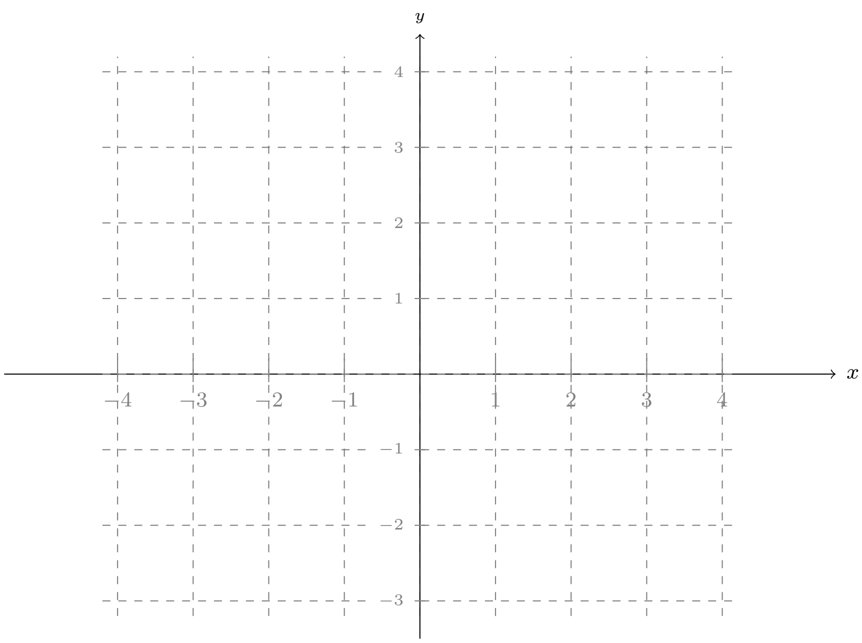
(ii) $\lim_{x \to -2^+} f(x) = \infty$

(iii) $\lim_{x \to 3^+} f(x) = -1$

(iv) $\lim_{x \to 3^-} f(x) = 3$

(v) $f(3) = 2$

(vi) Domain is $(-\infty, \infty)$.



GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

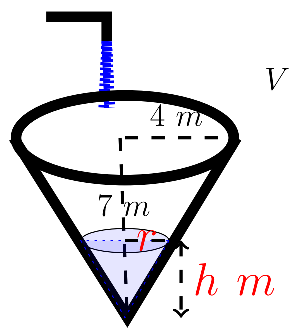
Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

4. **Background Story:** In industry, liquids are kept in different shape reservoirs and access to them brings up the rate of change. The next problem is shadowing that. Please note that the height and radius of the water are connected. Increase or decrease of one causes increase or decrease in the other respectfully. To find how they are related, use properties of similar triangles.

   Questions: Water is being pumped into a tank shaped like a cone. The tank has height 7 meters and the diameter at the top is 8 meters.

   $$V = \frac{\pi r^2 h}{3}$$

   (A) *(0.5 points)* Use the similarity of the two triangles to relate r (the radius of top of water in meters), h (the height of water in meters). Then write r as a function of h.

   (B) *(1 point)* Express the volume of the water in the tank as a function of the height of water, h, in meters. Make sure to simplify.

   (C) *(1.5 points)* What is the **average rate of change of the volume** when the height of the water is changing from $h_1 = a\ m$ to $h_2 = b\ m$? Make sure to simplify.

   Hint: $b^3 - a^3 = (b-a)(b^2 + ab + a^2)$

   (D) *(0.5 points)* What is the **average rate of change of the volume** when the height of the water increases from $h_1 = 3\ m$ to $h_2 = 5\ m$?