Week 3-Lab 2: Worksheet 4: Sections 2.3, 2.4 and 2.5

They asked: "Why is the $\lim\limits_{x\to a}$ *notation so important?" I said: "Why is* $\dfrac{(x-1)(x+1)}{x-1} \neq x+1$*?"*
They said: "Why so much algebra?" I said: " Because you need it in calculus II."

Short Descriptions and Formulas

Basic Limit Laws

Assume that $\lim\limits_{x\to c} f(x)$ and $\lim\limits_{x\to c} g(x)$ each **exist**.

Identity and Constant Laws $\lim\limits_{x\to c} x = c$ $\lim\limits_{x\to c} 1 = 1$

Sum Law $\lim\limits_{x\to c}(f(x)+g(x)) = \left(\lim\limits_{x\to c} f(x)\right) + \left(\lim\limits_{x\to c} g(x)\right)$

Constant Multiple Law $\lim\limits_{x\to c}(kf(x)) = k\left(\lim\limits_{x\to c} f(x)\right)$

Product Law $\lim\limits_{x\to c}(f(x)g(x)) = \left(\lim\limits_{x\to c} f(x)\right)\left(\lim\limits_{x\to c} g(x)\right)$

Quotient Law If $\lim\limits_{x\to c} g(x) \neq 0$, $\lim\limits_{x\to c}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{\lim\limits_{x\to c} f(x)}{\lim\limits_{x\to c} g(x)}$

Power Law If n is an integer, $\lim\limits_{x\to c}(f(x)^n) = \left(\lim\limits_{x\to c} f(x)\right)^n$

Continuity at a Point

A function f is **continuous** at the point $(c, f(c))$ if

$$\lim_{x\to c} f(x) = f(c)$$

A few immediate consequences of being continuous are

(i) The limit at c, $\lim\limits_{x\to c} f(x)$, exists.

(ii) The actual value, $f(c)$, exists.

(iii) There is a value L where $\lim\limits_{x\to c} f(x) = L = f(c)$.

Elementary Functions and Continuity

Elementary Functions are continuous on their domain.

Continuous Functions and Limits

Limit of Function is computed by direct sub. That is, for continuous function the predication is the actual value of the function.

Indeterminate Forms are called indeterminate because they represent limits which may or may not exist and may be equal to any value. The form itself does **not** indicate the value of the limit. There are 7 indeterminate forms:

$$\frac{0}{0} \qquad \pm\frac{\infty}{\infty} \qquad \infty-\infty \qquad \pm\, 0\cdot\infty \qquad 1^{\infty} \qquad 0^{0} \qquad \infty^{0}$$

Please write Form before writing any of those down. Never equate the limit to any of these.

Some Methods of Computing Limits That We Learned

Direct Substitution: Good for elementary functions on their domain and for other continuous functions.

Simplification: So far we used them for indeterminate forms $\frac{0}{0}$.

Conjugation: So far we used them for indeterminate forms $\frac{0}{0}$ with even roots.

The Squeeze Theorem: So far we only used them for functions with infinite oscillation.

Discontinuities

Jump discontinuity: When Limit from right $\neq$ limit from left at a point but they both exist.

Infinity: When one or two of the one-sided limits tend to infinity

Removable: When the limit exists but the value of the function is different at the point or does not exsit.

Non-decaying Fast Oscillation: We only seen $f(x) = \sin\left(\frac{1}{x}\right)$.

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Questions:**

   (A) Restrict the domain to makes $\dfrac{(x-1)(x+1)}{x-1} = x+1$ a correct statement.

   (B) Tell your friends why $\lim\limits_{x\to a} \dfrac{(x-1)(x+1)}{x-1} = \lim\limits_{x\to a} x+1$ is correct for all x-values even though $\dfrac{(x-1)(x+1)}{x-1} \neq x+1$ on entire $\mathbb{R}$.

2. **Background Story:** Sometimes we need to give a counter example.

   Questions:

   (A) What is the type of discontinuity where $\lim\limits_{x\to -2^-} f(x)$ and $\lim\limits_{x\to -2^+} f(x)$ both exist, but $\lim\limits_{x\to -2} f(x)$ does not exist? (Circle i, ii, or iii.)

   (i) Jump discontinuity (ii) Removable discontinuity (hole) (iii) Infinite discontinuity

   (B) Give an example where $\lim\limits_{x\to -2} \Big(f(x)\cdot g(x)\Big)$, $\lim\limits_{x\to -2^-} f(x)$ and $\lim\limits_{x\to -2^+} f(x)$ exist, but neither $\lim\limits_{x\to -2} f(x)$ nor $\lim\limits_{x\to -2} g(x)$ exists. (Circle all correct options.)

   (i) $f(x) = \begin{cases} 1 & x \le -2 \\ -1 & x > -2 \end{cases}$ and $g(x) = \begin{cases} -1 & x < -2 \\ 0 & x = -2 \\ 1 & x > -2 \end{cases}$.

   (ii) $f(x) = x+2$ and $g(x) = \dfrac{1}{x+2}$

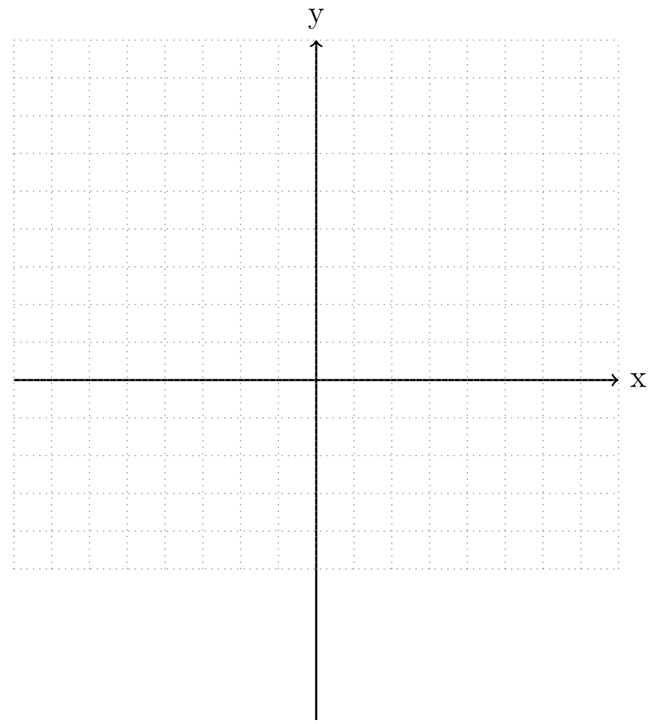
   (iii) $f(x) = \begin{cases} 1 & x < -2 \\ -1 & x \ge -2 \end{cases}$ and $g(x) = \begin{cases} -1 & x < -2 \\ 1 & x \ge -2 \end{cases}$

3. **Background Story:** The methods of Direct Subtitution, Simplification, Conjugation and squeeze theorem work for many elementary functions within their domain or outside their domain. Now what if the function is piecewise-defined? In that case you still can use those methods within each piece and for one-sided limits for each cut-off point.

Questions:

Let

$$f(x) = \begin{cases} -x-4 & \text{when } x < -2 \\ x & \text{when } -2 \le x < 3 \\ 3x-6 & \text{when } 3 < x \le 5 \\ \ln(x-5) & \text{when } 5 < x \end{cases}$$



(A) Use a function, according to the Background story, to fill in the first blank in each line. Then compute the limit:

$$\lim_{x \to -3} f(x) = \lim_{x \to -3} \boxed{} =$$

$$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} \boxed{} =$$

$$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} \boxed{} =$$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \boxed{} =$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \boxed{} =$$

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} \boxed{} =$$

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} \boxed{} =$$

(B) Identify the discontinuities of f as removable, jump or infinite.

(C) Sketch a graph of f(x) and check your answers from above.

4. **Background Story:** When using the limit laws, remember that the limit values get substituted and not the actual value of the function. In case of elementary functions these two are the same but they may not be in general. Be careful which one you are using.

Questions: Consider function f and g with the following information.

$\lim_{x\to-2} f(x) = 6$	$f(-2) = 6$	$\lim_{x\to 6} f(x) = -3$	$f(6) = -3$
$\lim_{x\to-2} g(x) = -1$	$g(-2) = -1$	$\lim_{x\to 6} g(x) = 3$	$g(6) = 3$

Now evaluate the following limits or say that they can not be computed (CBC).

(A) $\lim_{x\to-2} \dfrac{xf(x) - 2}{g(x) - 2} =$

(B) $\lim_{x\to-2} f(x)g(-3x) =$

(C) $\lim_{x\to 4} \dfrac{xf(x)}{g(x) - 3} =$

(D) $\lim_{x\to-2} f(x^2 + 2) =$

GroupWork Rubrics:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2^{nd} lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

5. **Background Story:** When seeing a limit where the rule of function is elementary, first attempt should be attempting to plug in the x-value. If you get a number, then you used direct substitution. If the result is of indeterminate form, then use simplification or conjugation. Do not use other methods we have not learned yet.

   Questions: Evaluate the following limits using the techniques introduced in sections 2.3, 2.4, and 2.5.

   (A) *(0.5 points)* $\lim\limits_{x\to -4} \dfrac{2x^2-6x+8}{x^2+16}$

   (B) *(1 point)* $\lim\limits_{x\to 3} \dfrac{\frac{1}{x}-\frac{1}{3}}{x-3}$

6. **Background Story:** Each of the limits is a limit of an average rate of change. That is, they are either $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ or $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$. Also, we have not used conjugates so far so now it is the time.

   Questions: *(2 points)* ① **Evaluate** each indeterminate form limit. ② Compare the limit to $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$; find the function $f(x)$ and the point a for this limit.

$$\lim_{x \to -2} \frac{\sqrt{x^2 + 12} - 4}{x + 2}$$

$f(x) =$

$a =$