Week 4-Lab 1:Worksheet 5: Sections 2.5 and 2.7

They asked: "Why is $\lim\limits_{x\to\pm\infty}$ *notation so important?" I said: Why is* $\dfrac{x+1}{4x+5} \neq \dfrac{1}{4}$*?"*

Short Descriptions and Formulas

Section 2.5 is only covered as a review:

We covered it in the last worksheet. We just want you to review before the quiz.

Limits at Infinity:

Vertical Asymptotes (When output tends to $\pm\infty$):
The line $x = a$ is a **vertical asymptote** of the curve $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x\to a} f(x) = \pm\infty \qquad \lim_{x\to a^+} f(x) = \pm\infty \qquad \lim_{x\to a^-} f(x) = \pm\infty$$

Horizontal Asymptotes (When input tends to $\pm\infty$):
The line $y = L$ is a **horizontal asymptote** of the curve $y = f(x)$ if either

$$\lim_{x\to\infty} f(x) = L \qquad \text{or} \qquad \lim_{x\to-\infty} f(x) = L.$$

Important Property If n is a positive rational number, then

$$\lim_{x\to\infty} \frac{1}{x^n} = 0 \qquad\qquad \lim_{x\to-\infty} \frac{1}{x^n} = 0$$

(if n has odd denominator, otherwise DNE)

Rational Functions:

$$\lim_{x\to\pm\infty} \frac{a_n x^n + \ldots + a_1 x + a_0}{b_m x^m + \ldots + b_1 x + b_0} = \begin{cases} 0 & \text{if } n < m \\ \dfrac{a_n}{b_n} & \text{if } n = m \\ \pm\infty & \text{if } n > m \end{cases}$$

Indeterminate Forms are called indeterminate because they represent limits which may or may not exist and may be equal to any value. The form itself does **not** indicate the value of the limit. There are 7 indeterminate forms:

$$\frac{0}{0} \qquad \pm\frac{\infty}{\infty} \qquad \infty - \infty \qquad \pm\, 0\cdot\infty \qquad 1^\infty \qquad 0^0 \qquad \infty^0$$

Please write "FORM" before writing any of those down. Never equate the limit to any of these.

Some Methods of Computing Limits That We Learned

Direct Substitution:
Good for elementary functions on their domain and for other continuous functions.

Simplification: forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$.

Conjugation: forms $\frac{0}{0}$ and $\infty - \infty$.

The Squeeze Theorem: Oscillations

1. **Background Story:** These questions are all from Section 2.5. One reason that the conjugate method of Section 2.5 is given is here is so you can compare it to conjugate method for Section 2.7.

   Questions:

   Evaluate the following limits using the techniques introduced in sections 2.3, 2.4, and 2.5.

   (A) Using conjugation when the limit form is $\frac{0}{0}$.

   i. Evaluate $\lim_{h\to 0} \frac{\sqrt{a+3h}-\sqrt{a}}{h}$ using multiplication by conjugate.

   ii. What factor did you simplify in this process?

   (B) Using piecewise-defined functions when function contains absolute values with limit of the form $\frac{0}{0}$.

   i. Fill in the blank: $|1-x| = \begin{cases} \boxed{} & \text{when } x \geq \boxed{} \\ \boxed{} & \text{when } x \leq \boxed{} \end{cases}$

   ii. Fill in the blank: $\frac{7-7x}{|1-x|} = \begin{cases} \frac{7-7x}{\boxed{}} & \text{when } x > \boxed{} \\ \frac{7-7x}{\boxed{}} & \text{when } x < \boxed{} \end{cases}$

   iii. Evaluate $\lim_{x\to 1^+} \frac{7-7x}{|1-x|}$ using piecewise-defined nature of the function.

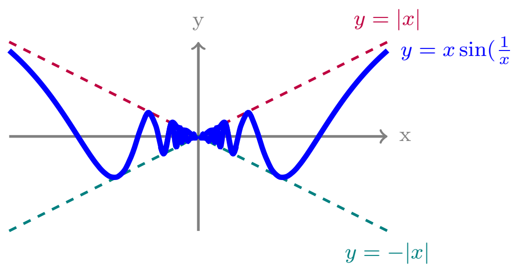
2. **Background Story:** One more problem of Section 2.5. Squeeze theorem turns out to be really handy tool in finding limits at infinity. Get some practice at it before Calculus II.

   (A) Fill in the blank $\boxed{} \leq \sin(x) \leq \boxed{}$.

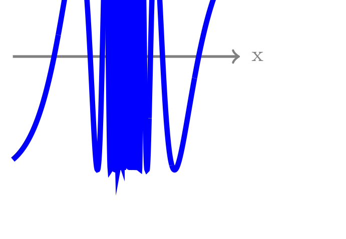
   (B) Fill in the blank $\boxed{} \leq \sin\left(\frac{1}{x}\right) \leq \boxed{}$ for all x in $(-\infty, 0) \cup (0, \infty)$.

   (C) Fill in the blank $\boxed{} \leq x\sin\left(\frac{1}{x}\right) \leq \boxed{}$ for all x in $(-\infty, 0) \cup (0, \infty)$.

   (D) Show that $g(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is continuous, using the Squeeze Theorem. (Click here for a visual explanation.)



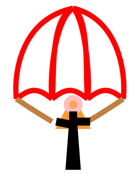
   (E) Can you use Squeeze theorem to compute $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$? Does the limit exist?



3. **Background Story:** We are interested the eventual output of a model. Every time you compute a $\lim_{t\to\infty} f(t)$ you are computing that eventual output.

   Questions:

   Velocity of skydiver at time t in meters per second, $v(t)$, is modeled by $v(t) = 50e^{-0.1t} - 50$.

   (A) Compute their eventual velocity.

   (B) What is the name of the eventual velocity in real life? (Guess!)

   (C) What is the velocity of them after $t = 50$ seconds? How close is that to the eventual velocity?

4. **Background Story:** Whatever I say about usefulness of exponential functions is not going to be enough. Learn about computing limits as $x \to \pm\infty$.

   Questions:

   (A) Evaluate each of these limits by looking at the horizontal asympototes and end-behavior of the graph.

   (i) $\lim_{x\to\infty} e^x = \infty$ (ii) $\lim_{x\to\infty} e^{-x} =$ (iii) $\lim_{x\to-\infty} e^x =$ (iv) $\lim_{x\to-\infty} e^{-x} =$

   (B) Find the horizontal asymptote(s), if any exist, for the following function:

   $$C(x) = \frac{-2e^x}{3 + 5e^{-x}}$$

5. **Background Story:** Some of you may know L'Hospital's Rule for finding limits. It turns out that L'hospital's rule is not the best method in computing the limits of square root functions. We get back to you on that in Section 4.5.

Questions:

(A) Find the horizontal asymptotes, if any exist, for the following function: $A(x) = \dfrac{\sqrt{49x^{10} - x}}{4x^5 + 3}$ Video: https://mediahub.ku.edu/media/t/1_j0lyjspo

(B) Find the horizontal asymptotes, if any exist, for the following function:

$$B(x) = \sqrt{9x^2 - 6x} - 3x$$

Video: https://mediahub.ku.edu/media/t/1_eygpfs1x

GroupWork Rubrics:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

6. *(3 points)* **Background Story:** Pay attention to the difference between this question and the problem before in the group work.

   Questions: Find the horizontal asymptotes, if any exist, for the following function:

$$C(x) = \sqrt{9x^2 - 6x} + 3x$$

7. **Background Story:** A limit of composition of continuous functions can be done by finding the limit of inside function then finding the limit of outside function.

   (A) *(1 point)* Use the end behaviors of the polynomial $x^8 + x^5$ to compute $\lim_{x \to -\infty} x^8 + x^5$.

   (B) *(1 point)* Evaluate $\lim_{x \to -\infty} \arctan\left(x^8 + x^5\right)$.

8. For integer n,

   (A) *(1 point)* Evaluate $\lim_{n \to \infty} \left(\frac{5}{8}\right)^n$ and $\lim_{n \to \infty} \left(\frac{-3}{8}\right)^n$.

   (B) *(1 point)* Evaluate $\lim_{n \to \infty} \frac{(5)^n + (-3)^n}{(8)^n}$.