Week 6-Lab 1: Worksheet 7: Section 3.3 and 3.7

She said: " I was solving the questions correctly for my groups but every time, they preferred his answers over mine. He was giving exactly same answers." I said: " I am so sorry!"; I asked my daughter: "What did you do when this happened to you?" She said: "I had an ally. He said that I did a better job answering the questions than he did. That reduced the pain." I am asking you: Be an ally!

Differentiating a Product of Functions

Suppose that $f(x)$ and $g(x)$ are two differentiable functions.

(I) The product $fg(x)$ is differentiable at $x = a$.

(II) $(fg)'(a) = f(a)g'(a) + g(a)f'(a)$. or equivalently

$$\frac{d}{dx}\left(fg(x)\right)\Big|_{x=a} = \left(f(x)\frac{dg}{dx} + g(x)\frac{df}{dx}\right)\Big|_{x=a}$$

(These last two equations say the same thing in Lagrange and Leibniz notation respectively.)

Differentiating a Quotient of Functions

If $f(x)$ and $g(x)$ are differentiable at $x = a$ and $g(a) \neq 0$, then $\left(\frac{f}{g}\right)(x)$ is differentiable at $x = a$ and

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}.$$

Exponential Derivatives:

$$(1)\frac{d}{dx}e^x = e^x \qquad (2)\frac{d}{dx}b^x = b^x \ln(b) \quad b > 0$$

The Chain Rule

If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $H = f \circ g$ defined by $H(x) = f(g(x))$ is differentiable at x, and its derivative is

$$H'(x) = f'(g(x))\, g'(x)\ .$$

$$\underbrace{f'}_{\text{Outside Derivative}} \quad \underbrace{(g(x))}_{\text{(Inside Untouched)}} \times \underbrace{g'(x)}_{\text{Inside Derivative}}$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}\ .$$

Some Gateway Instructions:

- Do not simplify. Example: $(-2)(3)x^2$ counts as $-6x^2$
- Write product rule in parenthesis: $(\quad)(\quad)+(\quad)(\quad)$
- Write Quotient rule in parenthesis: $\dfrac{(\quad)(\quad)-(\quad)(\quad)}{(\quad)^2}$
- Write chain rule in parenthesis: $(\quad)(\quad)\cdots(\quad)$

Gateway Videos:

11. Question 11: https://mediahub.ku.edu/media/MATH+125+-+011/1_zeoepv6f
12. Question 13: https://mediahub.ku.edu/media/MATH+125+-+012/1_0tqw1fru
13. Question 14: https://mediahub.ku.edu/media/MATH+125+-+013/1_kc1ru0t1
14. Question 15: https://mediahub.ku.edu/media/MATH+125+-+014/1_0nw0apbv
15. Question 16: https://mediahub.ku.edu/media/MATH+125+-+015/1_0yz7mvnc
16. Question 19:https://mediahub.ku.edu/media/MATH+125+-+016/1_k4mwh16d
17. Question 20: https://mediahub.ku.edu/media/MATH+125+-+017/1_ojkscffi
18. Question 21: https://mediahub.ku.edu/media/MATH+125+-+018/1_haiu706o
19. Question 22: https://mediahub.ku.edu/media/MATH+125+-+019/1_1gdh74uv
20. Question 23: https://mediahub.ku.edu/media/MATH+125+-+020/1_yofit0kp
21. Question 24: https://mediahub.ku.edu/media/MATH+125+-+021/1_lpzmhx05
22. Question 25: https://mediahub.ku.edu/media/MATH+125+-+022/1_95zwpz25
23. Question 023: https://mediahub.ku.edu/media/MATH+125+-+023/0_kbnphgru
24. Question 024: https://mediahub.ku.edu/media/MATH+125+-+024/0_v975r2mw
25. Question 025: https://mediahub.ku.edu/media/MATH+125+-+025/0_3pjqmjft
26. Question 026: https://mediahub.ku.edu/media/MATH+125+-+026/0_yecpxj2r
27. Question 027: https://mediahub.ku.edu/media/MATH+125+-+027/0_7j4dx8ez
28. Question 028: https://mediahub.ku.edu/media/MATH+125+-+028/0_7qjw2ogg
29. Question 029: https://mediahub.ku.edu/media/MATH+125+-+029/0_xdxzrvtq
30. Question 030: https://mediahub.ku.edu/media/MATH+125+-+030/0_89ubmyxk

Group Work Portion of the Worksheet

Names: _____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. Practice Gateway exam. Take the derivative of each of the following functions.

Product Rule or Product Tricks:

(A) $f(x) = (x^2 + 2x + 5)(x^3 + 1)$

(B) $f(x) = x^{-\frac{1}{2}}\,(1 + x^2 + 3x)$

(C) $h(w) = \left(w^{-\frac{1}{3}} - 3w^6\right)(4w^2 - 2w + 7)$

(D) $F(x) = \left(3x^2 + \sqrt{7}\,x - \pi^2\right)\left(\dfrac{x^4}{3} - \dfrac{x^2}{\sqrt{10}}\right)$

Quotient Rule:

(E) $f(x) = \dfrac{x^4 - 3x^2 + 2}{x^2 - 2}$

(F) $f(x) = \dfrac{x^3 - 1}{\sqrt[3]{x}}$

(G) $m(y) = \dfrac{1 - 4y^2}{6y^2 + 1}$

2. **Background Story:** While you compute the derivative of a composite functions, there may be multiple layers that you have to mind or you may need to use other derivative rules such as sum/difference rule, product rule and quotient rule. Discuss what rule is being used and how you use the derivative of inside and outside.

   Questions: Let $f(x)$ and $g(x)$ be differentiable functions such that

   $$f(1) = -3 \qquad f'(1) = -2 \qquad g(1) = 2 \qquad g'(1) = 1$$

   For each of the following, calculate the value (if it can be done) from the information. Discuss with each other what derivative rules you used.

   (i) $\dfrac{d}{dx}\left(f\Big(3x - g(x)\Big)\right)\Big|_{x=1}$

   (ii) $\dfrac{d}{dx}\left(e^{f(x)g(x)}\right)\Big|_{x=1}$

   (iii) $\dfrac{d}{dx}\left(g\left(f\left(\dfrac{g(x)}{2}\right)+4\right)\right)\Big|_{x=1}$

3. (i) Use the **limit definition of derivative** to calculate the derivative of the below function at $a = 4$.

$$f(x) = \frac{1}{\sqrt{5x+3}}$$

(ii) Verify your answer in part (a) by taking the derivative using a chain rule.

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: _____

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

4. Compute the following derivatives.

(a) *(0.5 points)* $\dfrac{d}{dy}\left(3y^7 - 8e^y\right) =$

(b) *(0.75 points)* $\dfrac{d}{d\theta}\left(3\theta^7 e^\theta\right) =$

(c) *(0.75 points)* $\dfrac{d}{dx}\left(\left(3x^7 - 8x^4 + 5x^3\right)\left(e^x + 11\right)\right) =$

(d) *(0.75 point3)* $\dfrac{d}{dx}\left(\dfrac{3x^7 - 8x^4 + 5x^3}{32e^x + 15}\right) =$

(e) *(0.75 points)* Evaluate $\dfrac{d}{dx}\Big(\left(x^2 + 5\right)\sqrt[3]{x^2 + 7}\Big)$.