Week 6-Lab 2: Worksheet 8: Section 3.7 and 3.8

I said: "The paper gateway exam is just around the corner. Paper Gateway exam is on the first day of lab Week 8, the week after Midterm 1. This way we can also break the material covered so far into two pieces (Midterm 1 + Gateway exam.) Gateway exams cover the entire derivative rules. A good source for review material can be found on Canvas Syllabus module. The Gateway exams are pass or no pass exams but you will have many chances to take them on computer during the week 9 and 10. "

Exponential Derivatives:

$$(1)\frac{d}{dx}e^x = e^x \qquad (2)\frac{d}{dx}b^x = b^x \ln(b) \quad b>0$$

$$(1)\frac{d}{dx}e^{g(x)} = e^{g(x)}g'(x) \qquad (2)\frac{d}{dx}b^{g(x)} = b^{g(x)}\ln(b)g'(x) \quad b>0$$

Logarithmic Derivatives:

$$(1)\frac{d}{dx}\ln(x) = \frac{1}{x} \qquad (2)\frac{d}{dx}\log_b(x) = \frac{1}{x\ln(b)} \quad b>0$$

$$(1)\frac{d}{dx}\ln(g(x)) = \frac{g'(x)}{g(x)} \qquad (2)\frac{d}{dx}\log_b(g(x)) = \frac{g'(x)}{g(x)\ln(b)} \quad b>0$$

Laws of Exponents and Logarithm: Use these before taking derivative.

Let $a>0$ and $b>0$ and let x and y be real numbers. Then,

1. $b^x.b^y = b^{x+y}$
2. $\frac{b^x}{b^y} = b^{x-y}$
3. $(b^x)^y = b^{xy}$
4. $(ab)^x = a^x b^x$
5. $(\frac{a}{b})^x = \frac{a^x}{b^x}$

If $m>0$ and $n>0$ and $b>0$, $b\neq 1$, then

1. $\log_b(mn) = \log_b(m) + \log_b(n)$
2. $\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n)$
3. $\log_b(m^n) = n\log_b(m)$
4. $\log_b(1) = 0$
5. $\log_b(b) = 1$

Memorize the following trig derivatives for the gateway exams:

$$\frac{d}{dx}\big(\sin(x)\big) = \cos(x) \qquad \frac{d}{dx}\big(\cos(x)\big) = -\sin(x)$$

$$\frac{d}{dx}\big(\tan(x)\big) = \sec^2(x) \qquad \frac{d}{dx}\big(\cot(x)\big) = -\csc^2(x)$$

$$\frac{d}{dx}\big(\sec(x)\big) = \sec(x)\tan(x) \qquad \frac{d}{dx}\big(\csc(x)\big) = -\csc(x)\cot(x)$$

Implicit Differentiation:

- Take the derivative of both sides.
- Remember if y is an inside function and use $\frac{dy}{dx}$

$$\frac{d}{dx}(y) = \frac{dy}{dx} \qquad \frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx} \qquad \frac{d}{dx}(e^y) = e^y\frac{dy}{dx}$$

$$\frac{d}{dt}(x) = \frac{dx}{dt} \qquad \frac{d}{dt}(y) = \frac{dy}{dt} \qquad \frac{d}{dt}(y^n) = ny^{n-1}\frac{dy}{dt}$$

More Gateway Videos:

Question 041: https://mediahub.ku.edu/media/MATH+125+-+041/0_zq5bhltn
Question 042: https://mediahub.ku.edu/media/MATH+125+-+042/0_9h2pvvq3
Question 043: https://mediahub.ku.edu/media/MATH+125+-+043/0_b9lxrg4o
Question 044: https://mediahub.ku.edu/media/MATH+125+-+044/0_r94ubcu3
Question 045: https://mediahub.ku.edu/media/MATH+125+-+045/0_wt8tsmo1
Question 046: https://mediahub.ku.edu/media/MATH+125+-+046/0_c4h5ap3i
Question 047: https://mediahub.ku.edu/media/MATH+125+-+047/0_k0ma4trg
Question 048: https://mediahub.ku.edu/media/MATH+125+-+048/0_76fkwq26
Question 049: https://mediahub.ku.edu/media/MATH+125+-+049/0_88vtbpht
Question 050: https://mediahub.ku.edu/media/MATH+125+-+050/0_00gcit4p
Question 51: https://mediahub.ku.edu/media/MATH+125+-+051/0_13cjxvpg
Question 52: https://mediahub.ku.edu/media/MATH+125+-+052/0_7l00ng66
Question 53: https://mediahub.ku.edu/media/MATH+125+-+053/0_osyikih0
Question 54: https://mediahub.ku.edu/media/MATH+125+-+054/0_vp1ouisl
Question 55: https://mediahub.ku.edu/media/MATH+125+-+055/0_23idubl3
Question 56: https://mediahub.ku.edu/media/MATH+125+-+056/0_wvmcve7o
Question 57: https://mediahub.ku.edu/media/MATH+125+-+057/0_wih87lxk
Question 58: https://mediahub.ku.edu/media/MATH+125+-+058/0_bh6ifi6z
Question 59: https://mediahub.ku.edu/media/MATH+125+-+059/0_x5hjwz8d
Question 60: https://mediahub.ku.edu/media/MATH+125+-+060/0_6q5o2ao3
Question 71: https://mediahub.ku.edu/media/MATH+125+-+071/0_t1a7qzkw
Question 72: https://mediahub.ku.edu/media/MATH+125+-+072/0_30vbj5h6
Question 73: https://mediahub.ku.edu/media/MATH+125+-+073/0_2in2rm3v
Question 74: https://mediahub.ku.edu/media/MATH+125+-+074/0_r5fpc5po
Question 75: https://mediahub.ku.edu/media/MATH+125+-+075/0_0oa1lldf

Group Work Portion of the Worksheet

Names: _____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. Practice derivative of trigonometric functions. Evaluate the following derivatives.

   (a) $\left.\dfrac{d}{dx}\left(\cos\left(5\pi x\right)\right)\right|_{x=1}$

   (b) $w(t) = 17 - \dfrac{\cos(t)}{17}$

   (c) $h(a) = 3\sin(a) - \cos(a)$

   (d) $f(y) = \dfrac{\sin(y)}{y}$

   (e) $m(t) = t\tan(t)$

   (f) $p(u) = \dfrac{\tan(u)}{1-\tan(u)}$

2. Practice more gateway questions. Take the derivative of following Functions. Don't forget to convert $\sqrt[n]{\ }$ to exponents.

Chain Rule:

(a) $f(x) = \sqrt[3]{x^4 - 7x}$, Find $\left.\dfrac{df}{dx}\right|_{x=1}$

(b) $u(t) = \dfrac{1}{\sqrt{t^2 + 2t - 1}}$

(c) $h(s) = (1 + \sqrt{s})^{-\frac{1}{2}}$

(d) $f(x) = (x^2 + 1)^{-10}$

(e) $m(u) = \sqrt{1 + \sqrt{u}}$

Combination of Chain/product/Quotient Rules:

(f) $f(x) = \left(1 + (x^2 + 2)^{\frac{1}{2}}\right)^{\frac{1}{3}}$

(g) $h(w) = \left(1 + \sqrt{w^3 + 3}\right)^4$

(h) $g(t) = (t^3 - 1)^4 (1 + t + t^2)^{-4}$

(i) $h(s) = [(s + 2)^3 (2 - s)]^3$

(j) $h(s) = \sqrt{\dfrac{s^2 + s - 2}{s + 2}}$

(k) $f(x) = \dfrac{5 - x}{2(x - 2)^{\frac{5}{2}}}$

(l) $f(x) = \left(\dfrac{x - 3}{x^2 + 7}\right)^4$

(m) $g(u) = \dfrac{2u - 3}{\sqrt{u^2 - 3u + 4}}$

3. Practice more gateway questions. Evaluate the derivative of the following functions.

Exponential Rules:

(a) $f(x) = (x^2 + 3x)\, e^x$, find $\left.\dfrac{df}{dx}\right|_{x=0}$

(b) $f(x) = x^2 2^x$

(c) $f(x) = 3^{x^2+1}$

(d) $f(x) = \dfrac{e^{x^2}}{e^{x-1}}$

(e) $f(x) = \dfrac{e^{-x}}{x}$

(f) $f(x) = x^2 e^{-x}$

(g) $f(x) = e^{-\frac{1}{x^2}}$

(h) $f(x) = x^4 + 4^x$

(i) $f(x) = \left(\dfrac{1}{2}\right)^x$

(j) $f(x) = e^{\sqrt{x}+x^2+2}$

(k) $f(x) = \dfrac{1 + e^{2x}}{2 - e^{2x}}$

4. Practice more gateway questions. Evaluate the derivative of the following functions.

More Exponential and Logarithmic Rules:

(a) $f(x) = e^x \ln(x)$

(b) $f(x) = \ln(3xe^x)$

(c) $f(x) = \ln\left(\dfrac{x-1}{x^2+1}\right)$

(d) $f(x) = \ln\left(\dfrac{e^x}{1+e^x}\right)$

(e) $f(x) = x^2 \ln(2x) + x \ln(3x) + 4 \ln(x)$

(f) $f(x) = \ln\left(\dfrac{1}{x}\right) - \dfrac{1}{\ln(x)}$

(g) $f(x) = x \ln(\sqrt{x}) + \ln(x^{-2})$

(h) $f(x) = (\ln(7x))^{\frac{1}{2}}$

5. **Background Story:** When taking derivative implicitly, always be aware that the derivative of a function of dependent variable, y, with respect to dependent variable, x, always includes $\dfrac{dy}{dx}$.

Questions: Find $\dfrac{dy}{dx}$: $\sin(y) = e^{-\sqrt{x}}$.

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: _____

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

6. **Background Story:** Sometimes a quotient rule follows by chain rule(s).

   Questions: *(1.5 point)* Evaluate $\dfrac{d}{dy}\left(\dfrac{(y-1)^7}{(y^2+5y)^4}\right)$.

7. **Background Story:** Evaluate the implicit **derivative**:

   Questions: *(2 points)* Find $\dfrac{dz}{dx}$: $z^7 = e^{xz}$