Week 7-Lab 2: Worksheet 9: Sections 3.4, 3.5 and Gateway

They said: "I am so anxious! What do I do to pass the paper Gateway?" I said: "Print All-Gateway-Questions page. Do everyone of them. Look at the videos to make sure you have done them correctly. Math is only learned by doing."
I added: "In case, you didn't pass the paper gateway, go to computer lab in the Snow hall and retake the exam. Remember if you pass the exam by answering 8 or more questions correctly out of 10 questions on the exam. When you pass the exam, your gateway score is 100%. This boast your grade immediately."

Exponential Derivatives:

$$(1)\frac{d}{dx}e^x = e^x \qquad (2)\frac{d}{dx}b^x = b^x \ln(b) \quad b>0$$

$$(1)\frac{d}{dx}e^{g(x)} = e^{g(x)}g'(x) \qquad (2)\frac{d}{dx}b^{g(x)} = b^{g(x)}\ln(b)g'(x) \quad b>0$$

Logarithmic Derivatives:

$$(1)\frac{d}{dx}\ln(x) = \frac{1}{x} \qquad (2)\frac{d}{dx}\log_b(x) = \frac{1}{x\ln(b)} \quad b>0$$

$$(1)\frac{d}{dx}\ln(g(x)) = \frac{g'(x)}{g(x)} \qquad (2)\frac{d}{dx}\log_b(g(x)) = \frac{g'(x)}{g(x)\ln(b)} \quad b>0$$

Laws of Exponents and Logarithm: Use these before taking derivative.

Let $a>0$ and $b>0$ and let x and y be real numbers. Then,

1. $b^x.b^y = b^{x+y}$
2. $\frac{b^x}{b^y} = b^{x-y}$
3. $(b^x)^y = b^{xy}$
4. $(ab)^x = a^x b^x$
5. $(\frac{a}{b})^x = \frac{a^x}{b^x}$

If $m>0$ and $n>0$ and $b>0$, $b\neq 1$, then

1. $\log_b(mn) = \log_b(m) + \log_b(n)$
2. $\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n)$
3. $\log_b(m^n) = n\log_b(m)$
4. $\log_b(1) = 0$
5. $\log_b(b) = 1$

Derivative of Trig Functions

$$\frac{d}{dx}\big(\sin(x)\big) = \cos(x) \qquad \frac{d}{dx}\big(\cos(x)\big) = -\sin(x)$$

$$\frac{d}{dx}\big(\tan(x)\big) = \sec^2(x) \qquad \frac{d}{dx}\big(\cot(x)\big) = -\csc^2(x)$$

$$\frac{d}{dx}\big(\sec(x)\big) = \sec(x)\tan(x) \qquad \frac{d}{dx}\big(\csc(x)\big) = -\csc(x)\cot(x)$$

Gateway Instructions:

- Do not simplify. Example: $(-2)(3)x^2$ counts as $-6x^2$
- Write product rule in parenthesis: $\big(\quad\big)\big(\quad\big)+\big(\quad\big)\big(\quad\big)$

- Write Quotient rule in parenthesis: $\dfrac{(\quad)(\quad)-(\quad)(\quad)}{(\quad)^2}$
- Write chain rule in parenthesis: $(\quad)(\quad)\cdots(\quad)$
- Any exam has one question from each each category. That is 10 questions.
- Eight, nine or ten correct answers is a passing grade.
- No partial credits.
- No calculators.

The Distance, Velocity and Acceleration: Kinematics is all about calculus! (In fact, it was one of the original motivations for Newton to develop calculus as a separate branch of mathematics.)

Quantity	Symbol	Calculus	Units
Distance	$s(t)$		distance
Velocity	$v(t)$	$= s'(t)$	distance/time
Acceleration	$a(t)$	$= v'(t) = s''(t)$	distance/time^2
Jerk	$j(t)$	$= a'(t) = v''(t) = s'''(t)$	distance/time^3

We often use $h(t)$ (for height) instead of $s(t)$ when the motion is vertical.

Physics: Applications to Kinematics In physics, polynomials are used to model how gravity affects the height of a projectile. Gravity on Earth provides a constant acceleration of -9.8 m/sec^2 $\approx$ -32 ft/sec^2.

By the power rule, the degree of the height function $h(t)$ is two higher than the degree of acceleration $a(t)$. Since acceleration is constant, it has degree zero, and it follows that the height polynomial is quadratic:

$$\begin{aligned} h(t) = pt^2 + qt + r \quad &\Rightarrow \quad v(t) = h'(t) = 2pt + q \\ &\Rightarrow \quad a(t) = v'(t) = h''(t) = 2p \end{aligned}$$

What do p, q, r signify? Since $a(t) = -9.8$ we have $p = -4.9$. Also, $q = v(0) = v_0$ is the initial velocity of the object, and $r = h(0) = h_0$ is the initial position.

$$\boxed{h(t) = -4.9t^2 + v_0 t + h_0}$$

Implicit Differentiation with Respect to Time

$$\frac{d}{dt}(x) = \frac{dx}{dt} \qquad \frac{d}{dt}(y) = \frac{dy}{dt} \qquad \frac{d}{dt}(y^n) = ny^{n-1}\frac{dy}{dt}$$

All Gateway videos in Playlists:

1-20 : https://mediahub.ku.edu/playlist/dedicated/1_z7nvz23w/

21-40 : https://mediahub.ku.edu/playlist/dedicated/1_giyp93qu/

41-60 : https://mediahub.ku.edu/playlist/dedicated/1_q34w8k09/

61-80 : https://mediahub.ku.edu/playlist/dedicated/1_dyyselca/

81-100 : https://mediahub.ku.edu/playlist/dedicated/1_z7nvz23w/

More Gateway Videos, Individually:

Question 76: https://mediahub.ku.edu/media/MATH+125+-+076/0_r2eubdda
Question 77: https://mediahub.ku.edu/media/MATH+125+-+077/0_uu2lg3cz
Question 78: https://mediahub.ku.edu/media/MATH+125+-+078/0_v8mgbdeh
Question 79: https://mediahub.ku.edu/media/MATH+125+-+079/0_hq4e0tf0
Question 80: https://mediahub.ku.edu/media/MATH+125+-+080/0_tf7p4i13
Question 81: https://mediahub.ku.edu/media/MATH+125+-+081/0_e7g9nmt4
Question 82: https://mediahub.ku.edu/media/MATH+125+-+082/0_tvbmwxnu
Question 83: https://mediahub.ku.edu/media/MATH+125+-+083/0_cf3egrhf
Question 84: https://mediahub.ku.edu/media/MATH+125+-+084/0_jjep03m8
Question 85: https://mediahub.ku.edu/media/MATH+125+-+085/0_iufp0tyd
Question 86: https://mediahub.ku.edu/media/MATH+125+-+086/0_anz8d847
Question 87: https://mediahub.ku.edu/media/MATH+125+-+087/0_xxbr29pp
Question 88: https://mediahub.ku.edu/media/MATH+125+-+088/0_s44j10ku
Question 89: https://mediahub.ku.edu/media/MATH+125+-+089/0_bnne367s
Question 90: https://mediahub.ku.edu/media/MATH+125+-+090/0_k0o9a6qa
Question 91: https://mediahub.ku.edu/media/MATH+125+-+091/0_kh261rbc
Question 92: https://mediahub.ku.edu/media/MATH+125+-+092/0_av04ibup
Question 93: https://mediahub.ku.edu/media/MATH+125+-+093/0_793ucmku
Question 94: https://mediahub.ku.edu/media/MATH+125+-+094/0_5ld1ta58
Question 95: https://mediahub.ku.edu/media/MATH+125+-+095/0_za045tjs
Question 96: https://mediahub.ku.edu/media/MATH+125+-+096/0_biahhay0
Question 97: https://mediahub.ku.edu/media/MATH+125+-+097/0_le8dy859
Question 98: https://mediahub.ku.edu/media/MATH+125+-+098/0_kaqh4bz0
Question 99: https://mediahub.ku.edu/media/MATH+125+-+099/0_tc5lncst
Question 100: https://mediahub.ku.edu/media/MATH+125+-+100/0_6eu5sl0x

Group Work Portion of the Worksheet

Names: _____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. Practice more gateway questions. Take the derivative of the following functions.

Trig Rules:

(a) $g(v) = (\sin(v) - v\cos(v))^{-17}$

(b) $H(x) = \dfrac{\sin^2(x) + \cos(x)}{x^2 + x}$

(c) $f(t) = \left(1 + \sqrt{\sin(t)}\right)\left(1 - 2\sqrt{\cos(t)}\right)$

(d) $F(y) = \tan(17 + y)$

More Trig Rules:

(e) $h(r) = 4\cos^7(2 - 4r)$

(f) $l(y) = \sin\left(\sqrt{y}\right) + \sqrt{\sin(y)}$

(g) $m(x) = (\cos(1 - x^2))^{\frac{3}{2}}$

(h) $F(t) = 4t^3 - \dfrac{6}{t} + \dfrac{2}{\sin(3t^2 + 1)}$

(i) $h(x) = (x^2 + x - 1)^5 \sin(5x)$

(j) $f(s) = \dfrac{\tan(2s)}{\cos(1 - 2s)}$

(k) $h(y) = \sin(y^2)\sin^2(y)$

(l) $H(x) = \dfrac{\sin\left(\sqrt{3 - x}\right)}{\sqrt{\tan(4 - x)}}$

(m) $l(t) = \left(1 + (2t + 3\tan(4t))^{-\frac{1}{2}}\right)^{\frac{4}{3}}$

(n) $K(x) = \left(1 - \dfrac{\sin(\pi - x)}{\tan(\pi + x)}\right)^{\frac{2}{3}}$

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2. Practice more gateway questions. Take the derivative of the following functions.

More Trig Rules With Exponential and Logarithmic Rules:

(a) $F(t) = \sin(\tan(\pi t))$

(b) $G(x) = \tan(\cos(ex))$

(c) $m(b) = \cos\left(\sin\left(\sqrt{2}\, b\right)\right)$

(d) $k(s) = \cos(\pi \sin(1 - s^2))$

(e) $g(t) = \sin^2(t^2 + \tan(t))$

(f) $R(x) = 2\sin\left(\dfrac{1}{\cos(x)}\right)$

(g) $M(x) = \sqrt{\tan(\sin(4x))}$

(h) $F(y) = 1 + \sqrt{\pi + \cos(\sin(ey))}$

(i) $f(x) = \ln(\sin(x))$

(j) $f(x) = e^{3\cos(2x)}$

3. **Background Story:** The position function of a moving body indicates where that body is at time t. If the body is traveling in one direction, the total distance traveled during time $[t_{\text{start}}, t_{\text{end}}]$ will be $|s(t_{\text{end}}) - s(t_{\text{start}})|$. But if the direction of travel changes, the total distance is the sum of distances traveled in each direction.

   Questions: At time t seconds, the position of a body moving along the x-axis is

$$s(t) = 2t^3 - 15t^2 + 24t \quad \text{meters}$$

   (A) Find the equation of velocity and acceleration of the body.

   (B) Find the acceleration of the body each time the velocity is zero.

   (C) Sketch the graph of the velocity and acceleration of the body.

   In the next two parts we are trying to compute the total distance traveled during the time interval $[0, 2]$.

   (D) Use the graph of $v(t)$ to find an interval from $t = 0$ to $t = 2$ when the object is traveling in the positive x-direction and an interval(s) from $t = 0$ to $t = 2$ when it traveling in the negative direction.

   (E) Find the total distance traveled by the body from $t = 0$ to $t = 2$. *(Compute the total distance traveled using formula* $|s(t_{end}) - s(t_{start})|$ *on each of the intervals in Part (D). Then add the two total distances.)*

Click https://ggbm.at/ycjpbnam for a visual explanation of Total distance.

Video: https://youtu.be/y6IKijK-FC4

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: _____

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

4. A toy train is moving North and South on a **straight track** starting at the station at noon and finishing at 12 : 07 pm. At t minutes after noon it is $s(t) = t^3 - 18\,t^2 + 105\,t$ cm north of the station.

   Questions:

   (A) *(0.5 points)* Find the velocity of the train at any time t.

   (B) *(0.75 points)* When does the train stop and head back to the station?

   (C) *(0.5 points)* What is the furthest distance of the train from the station?

   (D) *(0.75 points)* What is the **acceleration** of the toy train when it starts to head back toward the station?

   (E) *(0.5 points)* What is the **position** of the toy train at 12 : 07?

   (F) *(1 point)* What is the total distance the train has traveled during that 7 minutes?

5. **Background Story:** In computing the derivative of a composite of two function at at instance, two rates at that instance is needed.

   Questions: *(3 points)* The volume of a sphere is given in terms of the radius as $V = \dfrac{4\pi r^3}{3}$. The radius is changing over time. At a particular instant, $r = 5$ and $\dfrac{dr}{dt} = -4$. What is $\dfrac{dV}{dt}$ at that instant?