Week 1-Lab 1 or 2: Worksheet 1: Vector Review

They asked: "How do I study for this course?"
*I said: "Learn the material every day."*
They asked:"How do I learn the material every day?"
In which I replied: "Read the appropriate section of the book before lecture, download the lecture notes, follow the lectures without distraction of your phones and other devices, work on worksheets before going to the labs, continue working on your worksheets in the labs to learn the material hands-on, and do the achieve assignments to practice the skills you learned, take the weekly online quiz at the end of each week, and retake the quiz if the first score is not desirable. Make sure to get help in our help room or at one of the free tutoring places on campus if a concept is difficult. Aforementioned are the most important part of learning. Then do all problems on review on your own the week before the exam to study."
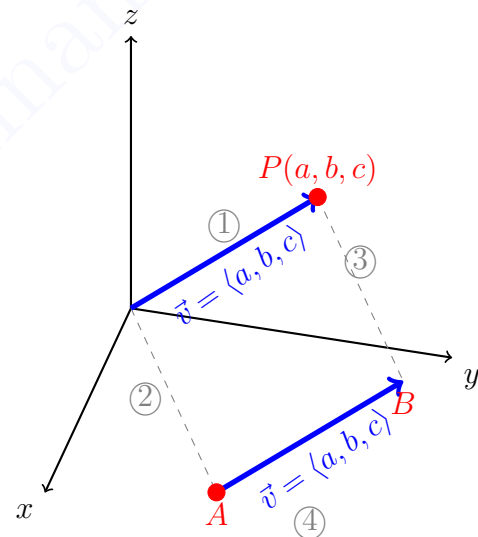
Short Descriptions and Formulas

How to draw a point in 3d:

How to draw a vector in 3D:
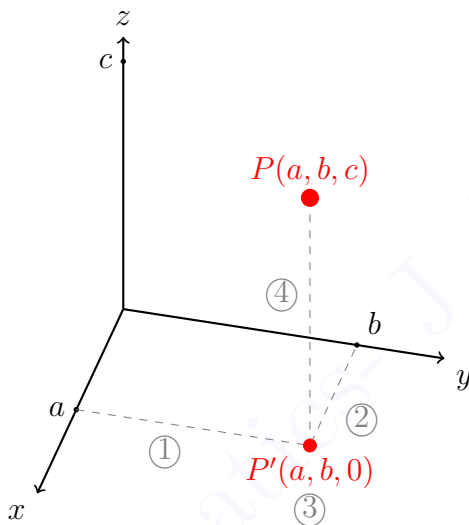
To draw point $P(a, b, c)$: ① Draw a line parallel to y-axis from point $(a, 0, 0)$. ② Draw a line parallel to x-axis from point $(0, b, 0)$. ③ Name the intersection of the two lines P'. ④ Draw a line parallel to z-axis from point $(a, b, 0)$ and mark c units to get to $P(a, b, c)$.
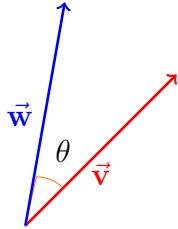
To draw the vector $\vec{v} = \langle a, b, c \rangle$ at an initial point A: ① First draw point (a, b, c) and connect $(0, 0, 0)$ to (a, b, c). ② Connect $(0, 0, 0)$ and A. ③ Draw a line segment parallel and the same length as to line segment in ② from $P(a, b, c)$. Denote the new end point B. ④ Draw the vector $\vec{v}$ from A to B.

Dot Product

The **dot product** of two vectors $\vec{\mathbf{v}} = \langle a_1, b_1, c_1 \rangle$ and $\vec{\mathbf{w}} = \langle a_2, b_2, c_2 \rangle$ is the scalar

$$\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = \|\vec{\mathbf{v}}\| \, \|\vec{\mathbf{w}}\| \, \cos(\theta)$$

where θ is the angle between the vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$.

- If $0 \leq \theta < \frac{\pi}{2}$ then $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} > 0$.
- If $\theta = \frac{\pi}{2}$ then $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = 0$.
- If $\frac{\pi}{2} < \theta \leq \pi$ then $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} < 0$.
- The angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ is $\arccos\left(\dfrac{\vec{\mathbf{v}} \cdot \vec{\mathbf{w}}}{\|\vec{\mathbf{v}}\| \, \|\vec{\mathbf{w}}\|} \right)$.

The Cross Product

The **cross product** of vectors $\vec{\mathbf{v}}, \vec{\mathbf{w}}$ in $\mathbb{R}^3$ is the vector

$$\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \left(\|\vec{\mathbf{v}}\| \, \|\vec{\mathbf{w}}\| \, \sin(\theta) \right) \vec{\mathbf{n}}$$

where:

(i) θ is the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$;

(ii) $\vec{\mathbf{n}}$ is the <u>unit</u> vector perpendicular to both $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$, given by the Right-Hand Rule.
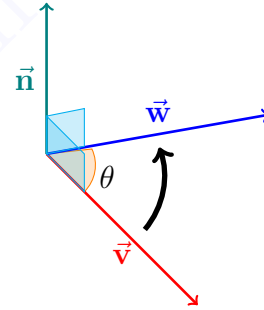
Cross Product Formula: $\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

The Formula for the Dot Product
Formula in $\mathbb{R}^2$: $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = a_1 a_2 + b_1 b_2$
Formula in $\mathbb{R}^3$: $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = a_1 a_2 + b_1 b_2 + c_1 c_2$

Dot and Cross Products

(1) The **dot product**, which takes two vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ (either <u>both in $\mathbb{R}^2$</u> or <u>both in $\mathbb{R}^3$</u>) and produces a *scalar* $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}}$.

(2) The **cross product**, which takes two vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ (<u>both in $\mathbb{R}^3$</u>) and produces a *vector* $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$.

It is very important to understand the **geometry** behind the dot and cross product, not just their formulas.

Group Work Portion of the Worksheet

Names: _____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Ice Breaker Background Story:** This will help you make friends in class and find people to with whom you can study do group works. Take turns to tell us

   (1) Your preferred name,   (2) What your major is,

   (3) Where you're from,   (4) Who you are,

   (5) Something we would not know just by looking at you.

2. **Background Story:** 3d designs are integrated parts of many fields of engineering. Understanding how to sketch points and lines in 3d is the first step in understanding how to sketch in 3d.

   Questions:

   (A) sketch the following vectors $\vec{i} = \vec{v} + \vec{w}$, $\vec{ii} = \vec{v} - \vec{w}$ and $\vec{iii} = -1.5\vec{v}$ using the parallelogram, triangle or other geometric properties. Label all items and type your name next to them.
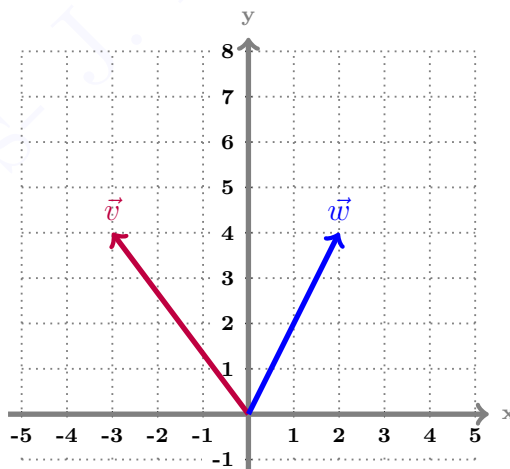

   (B) Sketch points $A = (-3, 2, 0)$ and $B = (-3, 2, 4)$, vector $\vec{v} = \langle -3, 2, 4 \rangle$ and a line ("l") through two points A and B. Label all items and type your name next to them.

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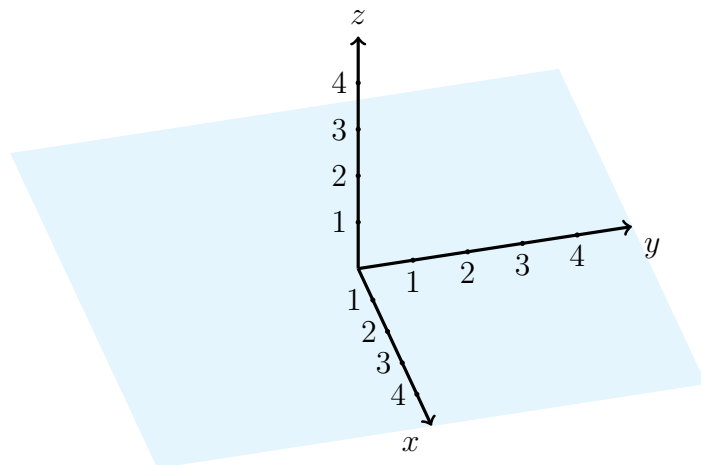
3. **Background Story:** The beauty of using the unit Cartesian basis $\vec{\mathbf{i}}$ and $\vec{\mathbf{j}}$ is that we can compute the vector sum, difference and scalar multiplication without sketching. In the next question, sketch and compute using Cartesian coordinates.

   Questions: Evaluate the following operations where the $\vec{v}$ and $\vec{w}$ are shown in the figure. Use the correct notation for the vectors, $\langle \ , \ \rangle$.
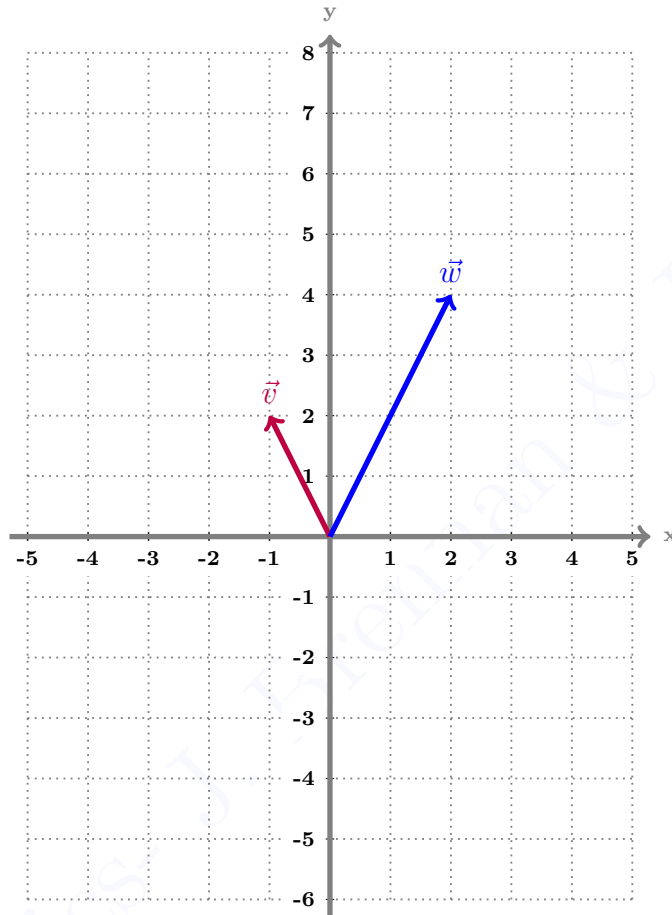
(i) $\vec{v} + \vec{w} =$

(ii) $\vec{v} - \vec{w} =$

(iii) $-2\vec{v} =$

(iv) $3\vec{v} =$

(v) $\vec{v} \cdot \vec{w} =$

(vi) $\vec{v} + 2\langle 1, 0 \rangle =$

4

4. **Background Story:** As you may have guessed, knowing in we have right angles in 3d is important. One application of dot product is in computing the angle between vectors given in Cartesian coordinates.

   Questions:

   (A) Is $\langle 0, 1, 3 \rangle$ orthogonal to $\langle -6, 1, 2 \rangle$? How do you know?

   (B) Is $\langle 0, 1, 3 \rangle$ orthogonal to $\langle 1, -6, 2 \rangle$? How do you know?

   (C) Find two unit vectors that are orthogonal to $\vec{v} = \langle \cos(t), \sin(t) \rangle$.
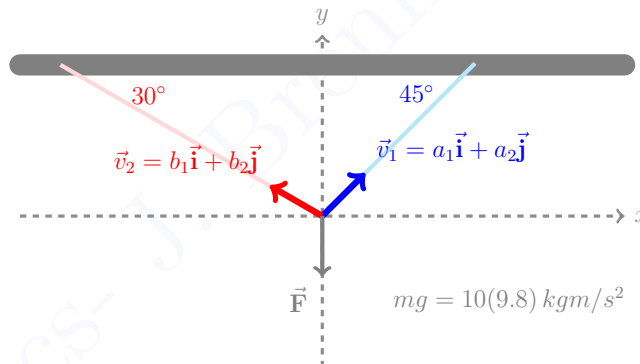
   (D) What is a directional vector of line $\langle 2 + 5t, 2, 7 + t \rangle$?

5. **Background Story:** In Physics and many fields of engineering, we find tensions in a 3d space. The solution to 3d problem is a generalization of the 2d problem.

   Questions: A $10\,kg$ traffic light is hanging from a rod by two ropes and is in equilibrium position as shown in this picture. We are interested in computing the tension on each rope.
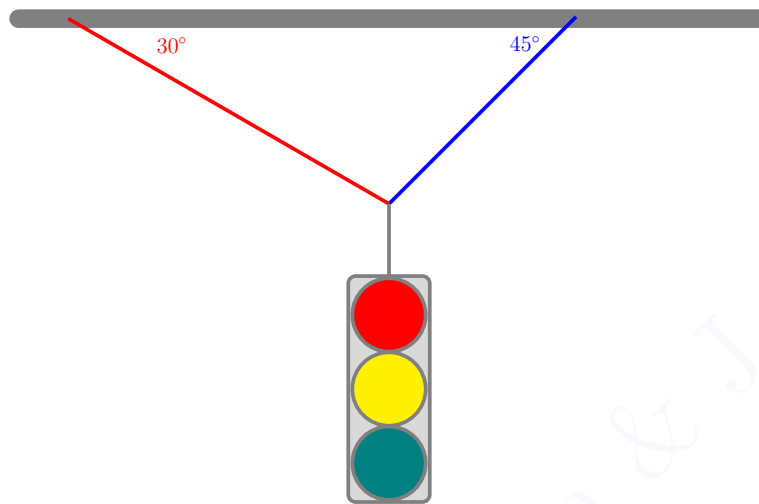
   (A) Represent the force of gravity as a vector in $\mathcal{R}^2$, denoted by $\vec{\mathbf{F}}$.

$$30° \qquad 45°$$
$$\vec{v}_2 = b_1\vec{\mathbf{i}} + b_2\vec{\mathbf{j}} \qquad \vec{v}_1 = a_1\vec{\mathbf{i}} + a_2\vec{\mathbf{j}}$$
$$\vec{\mathbf{F}} \qquad mg = 10(9.8)\,kgm/s^2$$

   (B) Use trigonometry to relate a_1 and a_2. (Give an equation) Use the trigonometry to relate b_1 and b_2. (Give another equation.)

   (C) "The traffic light is in equilibrium" means the net forces is zero ($\vec{v}_1 + \vec{v}_2 + \vec{\mathbf{F}} = \vec{0}$). What does it mean in terms of x-components of the vectors? (Give an equation.) What does it mean in terms of y-components of the vectors? (Give another equation.)

   (D) Use the four equations and solve for a_1, a_2, b_1, and b_2.

   (E) Rewrite $\vec{v}_1$ and $\vec{v}_2$ with numeric values you found in Part (D).

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: _____

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

1. **Background Story:** Well, dot products are good tools in computing angles but cross product are good tools in computing a vector orthogonal to two non-parallel vectors. This fact is helpful in finding equation of planes.

    Questions: Consider the two vectors $\vec{u} = \vec{j} + \vec{k}$ and $\vec{v} = 2\vec{i} - 3\vec{j} + \vec{k}$.

    (A) *(1.5 points)* Compute $\vec{u} \times \vec{v}$.

    (B) *(0.5 points)* How many unit vectors are orthogonal to $\vec{u}$? How about $\vec{v}$?

    (C) *(2 point)* Find two unit vectors that are orthogonal to both $\vec{u}$ and $\vec{v}$.

2. **Background Story:** In Chemistry, molecules have different symmetries. To understand how atoms interact in a molecule, first we learn the structure of the simple symmetrical solids, such as cube or a symmetric tetrahedron, the process that you learn with simplest Platonic solids can be repeated for more complicated molecules. So please remember that process is more important than result in the next question.

   Questions:

   (3 points) Use a dot product to find the angle between a main diagonal of a cube and a diagonal on a face of the cube.
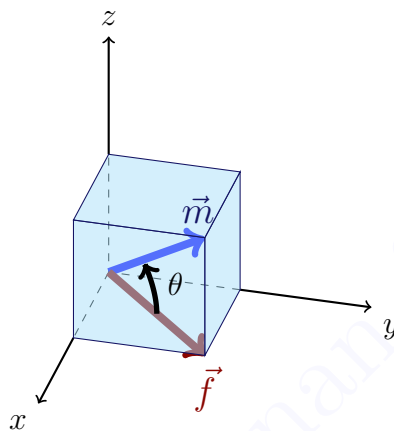
3. **Your Math Background and Group Work Requests:** Tell us a bit about your math background and whether you had a math course from us before.

   Write in a piece of paper and turn in to your instructor the answer to the Part (1)-(3).

   (1) What is the highest math class that you have taken?

   (2) Have you taken any math classes at KU?

   (3) Are you feeling comfortable with prerequisite material for this course?

   (4) Are you comfortable working in groups?

   (5) Are there people in class that you prefer to work with?

Math 127 and material, open letter to my students

Dear friends of Math 127,

We always receive reports of different websites that take our material and sell them for profit to students. They also pay small fees or allow free access to our students to obtain these material. Please note that this activity is illegal and is a form of intellectual property theft of your instructors. Please don't be tricked by these websites. We spend long hours outside our working hours to create these material so our students can use them for **free**.

We also receive reports of tutors on social media platforms such as Groupme who use our previous semester material and, instead of really helping you, they just give you the solution to material. This activity is academic dishonesty. Some of these tutors are paid by small groups and they seem to help everyone in the social media. Please only get help from those tutors if they are willing to be on the same social media platform(s) as your instructors or add us to the social media platform they are using. Please don't get tricked by their gimmicks to create a divide between you and your instructors. That "divide" is how they profit with very small amount of work. We are all about helping each other out in a way that helps you learn more. So please have us involved in your problem solving process.