Week 8-Lab 1: Worksheet 10: Section 15.1

They said:"Sometimes I feel sad/angry without a good reason." I said: " Part of adulthood is recognizing that those emotions can be overwhelming and some adults need to find help with those emotions. Make sure you get help if you need it. There are some resources on campus that can help you. Let us know if we can help too. Take care!"

Iterated Integrals: Fubini's Theorem

If $f(x,y)$ is continuous on a rectangle $R=[a,b]\times[c,d]$ then

$$\begin{aligned}\iint_R f(x,y)\,dA &= \int_{x=a}^{x=b}\left(\int_{y=c}^{y=d} f(x,y)\,dy\right)dx\\ &= \int_{y=c}^{y=d}\left(\int_{x=a}^{x=b} f(x,y)\,dx\right)dy\end{aligned}$$

The integrals on the right-hand-side are called **iterated integrals**.

To evaluate the iterated integral $\displaystyle\int_a^b\int_c^d f(x,y)\,dy\,dx$:

(i) Evaluate the inner integral for y by treating x as a fixed parameter. The result of $\displaystyle\int_c^d f(x,y)\,dy$ is a **function** of x.

(ii) Then evaluate the integral with respect to x.

Split Double Integrals

When a continuous function $f(x,y)=g(x)h(y)$ is a product of x-only and y-only on $R=[a,b]\times[c,d]$, then $\displaystyle\iint_R g(x)h(y)\,dA=\left(\int_b^a g(x)\,dx\right)\left(\int_c^d h(y)\,dy\right)$.

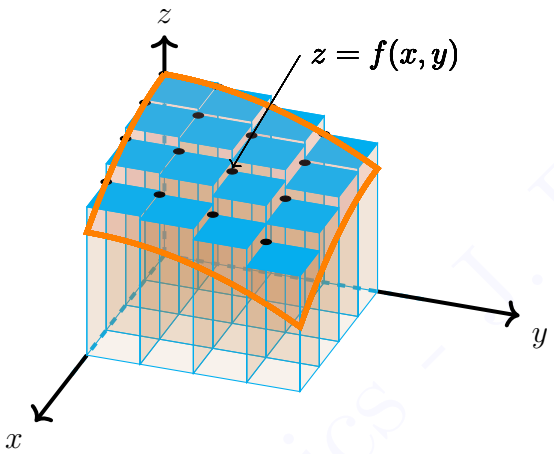
Group Work Portion of Worksheet

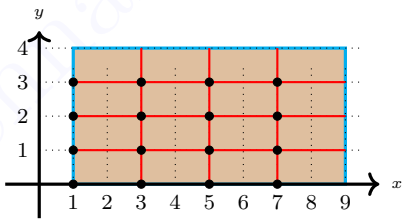
Names: _____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** You will see this problem on your Achieve assignments. You can discuss what it means when sampling points are lower left, upper left, lower right and upper right end points. This gives you a view of sample points in the domain and what it means to find the height of the rectangles at each sample point.

   Questions: Approximate $\int\int_{[1,9]\times[0,4]} f(x,y)\,dA$ as a Riemann sum $S_{4,4}$ using the lower left end points.





$f(1,4)=5$	$f(3,4)=5$	$f(5,4)=4$	$f(7,4)=3$	$f(9,4)=3$
$f(1,3)=6$	$f(3,3)=6$	$f(5,3)=5$	$f(7,3)=4$	$f(9,3)=3$
$f(1,2)=6$	$f(3,2)=6$	$f(5,2)=6$	$f(7,2)=5$	$f(9,2)=4$
$f(1,1)=7$	$f(3,1)=7$	$f(5,1)=6$	$f(7,1)=6$	$f(9,1)=5$
$f(1,0)=7$	$f(3,0)=7$	$f(5,0)=6$	$f(7,0)=6$	$f(9,0)=5$

2. **Background Story:** Discuss with your friends what are the inner and outer integral. Discuss when you can separate the integral and how to do that.

   Questions:

   (A) What is the inner integral in $\int_1^5 \int_2^8 \left(\frac{x}{y} + \frac{y}{x}\right) dy\, dx$.

   (B) Evaluate the iterated integral $\int_1^5 \int_2^8 \left(\frac{x}{y} + \frac{y}{x}\right) dy\, dx$.

   (C) True or False: $\int_1^5 \int_2^8 \left(\frac{x}{y}\right) dy\, dx = \left(\int_2^8 \left(\frac{1}{y}\right) dy\right)\left(\int_1^5 x\, dx\right)$. Explain your answer.

3. **Background Story:** This question is a review of solving inequalities from precalculus and is useful for section 15.2.

Questions:

(A) Shade the region bounded by $x = 0$, $y = \sqrt{x}$, and $y = 6$. (Graph the curves and use test points.)

(B) Shade the solution to $\begin{cases} 0 \le x \le 36 \\ \sqrt{x} \le y \le 6 \end{cases}$. (Use an arrow from the lower curve to higher curve/line.)

(C) Shade the solution to $\begin{cases} 0 \le x \le y^2 \\ 0 \le y \le 6 \end{cases}$. (Use an arrow from the lower curve to higher curve/line.)

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of Worksheet

Name: _____

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

4. **Background Story:** Use Fubini's theorem to compute $\int_1^3 \int_1^{18} x \sin(xy)\, dx\, dy$:

   Questions:

   (A) *(1.5 points)* Compute the **inner integral** of $\int_1^3 \int_1^{18} x \sin(xy)\, dx\, dy$.

   (B) *(1.5 points)* Compute $\int_1^{18} \int_1^3 x \sin(xy)\, dy\, dx$.

   (C) *(0.5 points)* Explain in a short sentence why you prefer to compute the integral using the order in Part (B).