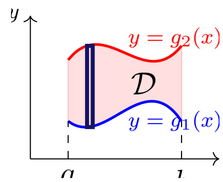
Week 8-Lab 2: Worksheet 11: Sections 15.2 and 15.3

They said: "Oh! That spring break is not coming early enough!" I said: "True my comrades! Is is right around the corner though! Hang in There!"

Simple Regions

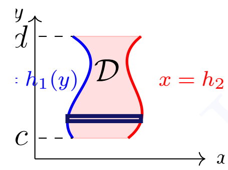
Vertically simple regions



Can be sliced into vertical strips each with constant x-coordinate

$$\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

Horizontally simple regions



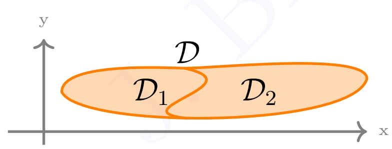
Can be sliced into horizontal strips each with constant y-coordinate

$$\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

Inner limits are functions of outer variable

Outer limits are constants

Integrating Over General Regions: The region of integration can be subdivided:



If $\mathcal{D}$ is the union of $\mathcal{D}_1$ and $\mathcal{D}_2$, where $\mathcal{D}_1$ and $\mathcal{D}_2$ don't overlap except on their boundaries, then

$$\iint_{\mathcal{D}} f(x,y)\,dA = \iint_{\mathcal{D}_1} f(x,y)\,dA + \iint_{\mathcal{D}_2} f(x,y)\,dA$$

When, how and why we reverse the order of integration:

When the region of integration is both vertically and horizontally simple. Which iterated integral should you use? **Whichever is more convenient.**

How? Find the inequalities describing the region of integration in the current order of integration and graph the region. Then use an appropriate arrow to reverse the order. Make sure that the two curves

For example:

(1) $\displaystyle\int_a^b \int_{y=g_1(x)}^{y=g_2(x)} f(x,y)\,dy\,dx$ means the region is $a \le x \le b$ and $g_1(x) \le y \le g_2(x)$. Graph the region and use a horizontal arrow to revers the order to a horizontally simple region.

(2) $\displaystyle\int_c^d \int_{x=h_1(y)}^{x=h_2(y)} f(x,y)\,dx\,dy$ means the region is $c \le y \le d$ and $h_1(y) \le x \le h_2(y)$. Graph the region and use a vertical arrow to revers the order to a vertically simple region.

Why? We often reverse the order in situations where the inner integral is either really difficult or does not have an elementary antiderivative. The reversing the order may result in a less complicated integral or elementary antiderivatives.

Non-elementary Antiderivatives:

What are elementary functions?

Those are the functions you know from your Precalculus and Calculus courses. (polynomials, exponential functions, logarithms, trigonometric functions, inverse trigonometric functions and any combinations of those under function operations.)

What functions have Antiderivatives?

According to the Fundamental Theorem of Calculus, all functions that are continuous on a closed and bounded interval have antiderivatives on that domain.

Can we find all antiderivatives using the methods we learned in Calculus?

Not all elementary functions have any elementary antiderivatives. That is, you can not find the antiderivative of all elementary functions using the methods we learned in calculus I and II even though they may exist. We can use numerical methods to evaluate an antiderivative at a point. These numerical methods often a result of Taylor polynomial approximation of the function.

What functions do not have elementary antiderivatives?

Here are some of the examples and there are infinitely more:

$$\int \sin(x^2)dx \int \cos(x^2)dx \int \sin(x^3)dx \int \ln(\ln x)dx \int e^{e^x}dx \int \frac{1}{\ln x}dx \int e^{x^2}dx \int x^{x^2}dx$$

How do I figure out if an integral is difficult to solve (using method of integration) or it can not be done in terms of elementary functions?

Searching it online sometimes pays off or go to Wolfram or other symbolic mathematical solvers and type in your integral to see if you get a non-elementary function or one of the terms is non-elementary. Below is an example.

What if the antiderivative is in Math 127 and is in a double integral?

Reverse the order of integration. That is all we are asking at this point. You learn about those antiderivative in other courses in more details.

Properties of Double Integrals

(1) If f and g are functions on $\mathcal{D}$, then

$$\iint_{\mathcal{D}} (f(x,y)+g(x,y))\,dA = \iint_{\mathcal{D}} f(x,y)\,dA + \iint_{\mathcal{D}} g(x,y)\,dA.$$

(2) If k is a constant, then $\displaystyle\iint_{\mathcal{D}} kf(x,y)\,dA = k\iint_{\mathcal{D}} f(x,y)\,dA.$

(3) If $f(x,y) \le g(x,y)$ for all (x,y) in $\mathcal{D}$, then

$$\iint_{\mathcal{D}} f(x,y)\,dA \;\le\; \iint_{\mathcal{D}} g(x,y)\,dA.$$

In this case, the volume of the solid between the graphs of f and g is

$$\iint_{\mathcal{D}} (g(x,y)-f(x,y))\,dA.$$

(4) The area of $\mathcal{D}$ is $\displaystyle\iint_{\mathcal{D}} 1\,dA.$

(5) The average value of $f(x,y)$ on $\mathcal{D}$ is

$$\overline{f} \;=\; \frac{\displaystyle\iint_{\mathcal{D}} f(x,y)\,dA}{\text{Area}(\mathcal{D})} \;=\; \frac{\displaystyle\iint_{\mathcal{D}} f(x,y)\,dA}{\displaystyle\iint_{\mathcal{D}} 1\,dA}.$$

(6) If m, M are constants and $m \le f(x,y) \le M$ for all (x,y) in $\mathcal{D}$, then

$$m\,(\text{Area}(\mathcal{D})) \le \iint_{\mathcal{D}} f(x,y)\,dA \le M\,(\text{Area}(\mathcal{D}))\,.$$

Triple integrals:

Use geometry of the picture to find two surfaces that bound the inner variable. This step does not require eliminating a variable between two equations.

After the inner integral is set. Find the reflection of the solid on the coordinate plane of the two variable. Treat the two outer integrals as a double integral on the region of reflection. This step may require eliminating a variable between two equations.

Group Work Portion of Worksheet

Names: ____

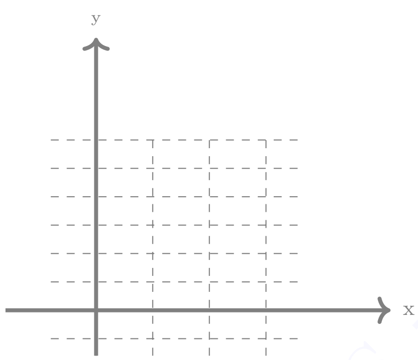
Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** Sometimes the order of integration as it results in an non-elementary antiderivative.

   Questions:

   (A) **Sketch** the region of integration for

   $$\int_0^{1/2} \int_{y^2}^{1/4} y\cos(24\pi x^2)\,dx\,dy$$

   (B) Discuss why you can not compute the double integral with the current order of integration.

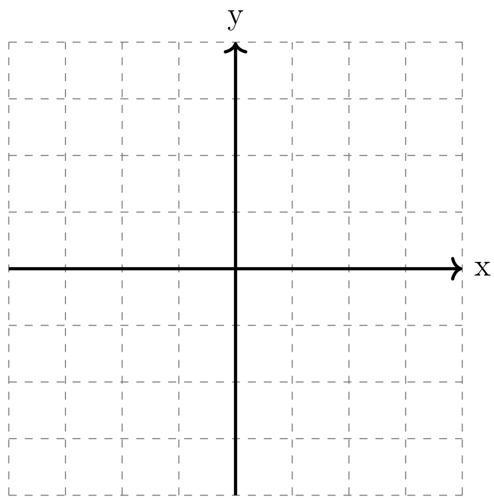
   (C) Evaluate the integral by **reversing** the order of integration:

   $$\int_0^{1/2} \int_{y^2}^{1/4} y\cos(24\pi x^2)\,dx\,dy$$

2. **Background Story:** A general region can be divided into simple regions with no overlapping interior. Then the sum of iterated integrals over those simple regions gives you the integral over that general region.

   Questions: Let $\mathcal{R}$ be the region bounded by the parallelogram with vertices $(1, 0)$, $(0, 3)$, $(-1, 0)$ and $(0, -3)$. Evaluate

$$\iint_{\mathcal{R}} e^y \, dA \, .$$



3. **Background Story:** Volumes can be evaluated with a double integral or a triple integral. Discuss those possibilities here.

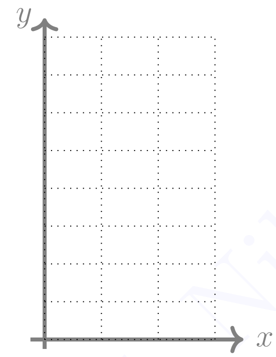
   Questions: Find the volume of the solid bounded by the surfaces $x^2 + y^2 = 16$, $y + z = 4$, and $z = 9$.

Link to the Geogebra sheet: https://www.geogebra.org/m/zvzaq8jw

4. We want to evaluate $\displaystyle\int\int_R \frac{2y}{x^2+y^2}\,dA$ where R is a triangle bounded by $y=x$, $y=4x$ and $x=2$:

Questions:

(A) Graph the region of integration.



(B) Is the region vertically simple, horizontally simple or both?

(C) Set up **an** iterated integral representing the double integral. *(Use a simple region.)*

(D) Solve the integral.

GroupWork Rubrics:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

5. *(3.5 points)* **Sketch** the region of integration and evaluate the integral by **reversing** the order of integration:

$$\int_0^{36} \int_{\sqrt{x}}^{6} \frac{1}{y^3+1}\, dy\, dx$$