Week 11-Lab 1: Worksheet 15: Section 13.1-13.2

They said: "How do I study for the exam?" I said: "Do each problem on the review, write all details, check your work against the solutions, then put the solutions away and redo the questions."

Vector-Valued Functions

A **scalar function** is a function whose output is a scalar.

A **vector function** (or **vector-valued function**) is a function whose output is a vector.

Derivatives of Vector-Valued Functions

Provided that $\vec{\mathbf{r}}'(t) \neq \vec{0}$, it is **tangent** to the curve parametrized by $\vec{\mathbf{r}}$.

Therefore, the tangent line to the curve of $\vec{\mathbf{r}}(t)$ at $t = a$ can be parametrized by the vector function

$$\vec{l}(t) = \vec{\mathbf{r}}(a) + t\,\vec{\mathbf{r}}'(a).$$

For example, a vector function $\vec{\mathbf{r}} : \mathbb{R} \to \mathbb{R}^3$ has the form

$$\vec{\mathbf{r}}(t) = \langle f(t), g(t), h(t)\rangle = f(t)\vec{\mathbf{i}} + g(t)\vec{\mathbf{j}} + h(t)\vec{\mathbf{k}}$$

where the independent variable t is a scalar in $\mathbb{R}$ and the dependent variable $\vec{\mathbf{r}}(t)$ is a vector in $\mathbb{R}^3$. The scalar functions f, g, and h are the **components** of the vector function $\vec{\mathbf{r}}$.

Example: Parametrize the intersection of the surfaces $y^2 + z^2 = 4$ and $x = 5y^2$.

This example is explained in details here: https://www.geogebra.org/m/bbpeqwbn

Arc Length Formula The arc length of the curve parametrized by $\vec{\mathbf{r}}(t)$ for $a \leq t \leq b$ is

$$\int_a^b \|\vec{\mathbf{r}}'(t)\|\,dt = \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}\,dt.$$

The Arc Length Function The length of the portion of the curve $\vec{\mathbf{r}}$ over the interval $[a, t]$ is

$$S(t) = \int_a^t \|\vec{\mathbf{r}}'(\tau)\|\,d\tau$$

The letter S is reserved for arc length. (Here τ is just a dummy variable.)
Note that $S(t)$ is a scalar function of t

Arc Length Parametrization: ①Find the arc length functions. ②Solve for t in (the original parameter) in terms of S. ③ Replace the original parameter in the original parameterization by $t(s)$.

Group Work Portion of Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. Find and sketch the projections of the curve on the three coordinate planes.

$$\vec{r}(t) = \langle \sin(t), t, 2\cos(t) \rangle$$

2. **Background Story:** Sketching the xy, yz and xz projections of the space curve helps you understand the curve. Try that for the following parametrizations so you can match them with the correct space curve.

Questions: Match the curve parameterized by each vector-valued function. (Enter I, II, III, IV, V, or VI.)

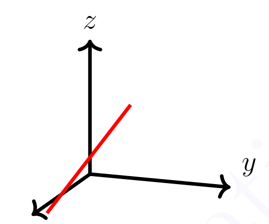
(a) $\mathbf{r}(t) = \langle \cos(t), \sin(t), \sin(4t) \rangle$

(b) $\mathbf{r}(t) = \langle t, \cos(t), \sin(t) \rangle$ and $t \geq 0$

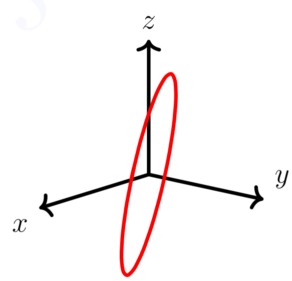
(c) $\mathbf{r}(t) = \langle \cos(t), \sin(t), 4\sin(t) \rangle$

(d) $\mathbf{r}(t) = \langle 3 + 2\cos(t), 1 + 4\cos(t), 2 + 5\cos(t) \rangle$

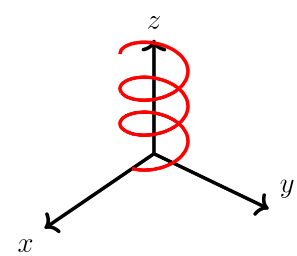
(e) $\mathbf{r}(t) = \langle \cos(t^3), \sin(t^3), t^3 \rangle$ and $t \geq 0$



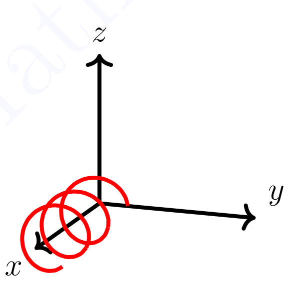
(I)



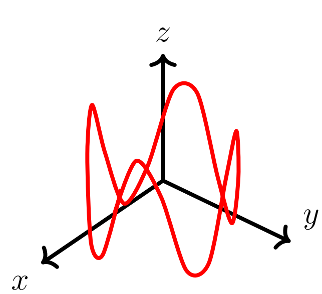
(II)



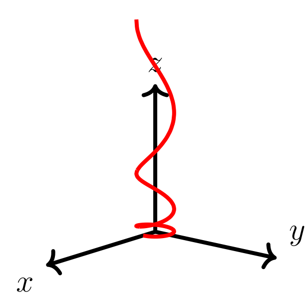
(III)



(IV)



(V)



(VI)

Video: https://youtu.be/G1AUgv2kzeA

3. **Background Story:** Parametrization of a curve in 3-D is not that different from parametrization in 2-D. In fact, we are asking you to try a 2-D parametrization for two variables and then solve for the last one. Also, Geogebra ia a fantastic free tool that you can use. https://www.geogebra.org/m/dnackfvt

   Questions: Find a parametrization of the curve that represents the curve of intersection of each pair of surfaces.

   (A)

$$\frac{x^2}{16} + \frac{z^2}{36} = 1 \qquad x + y = 5$$

   (B)

$$z = x^2 - y^2 \qquad x^2 + y^2 = 1$$

GroupWork Rubrics:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2^{nd} lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

4. *(3.5 points)* Find a vector equation for the tangent line to the curve of intersection of the surfaces at the point $(5, 12, 7)$.

$$x^2 + y^2 = 169 \qquad y^2 + z^2 = 193$$