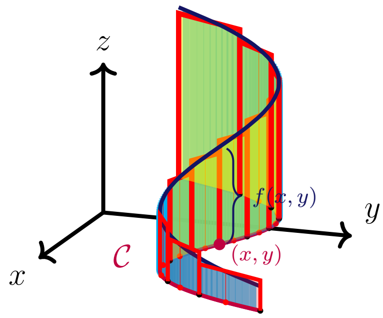
Week 13-Lab 1: Worksheet 18: Section 16.2

I said: " For us, it has been a year with post lock-down deficiencies and extreme reductions in resources. Despite it all, watching your progress keeps us going. Please continue the hard work! Don't slow down! Summer is around the corner! "

Scalar Line Integral:



The same formula works for curves in $\mathbb{R}^n$ (for $n = 2, 3, \dots$):

$$\int_{\mathcal{C}} f\,ds = \int_{\mathcal{C}} f(x_1,\dots,x_n)\,ds = \int_a^b f\big(\vec{\mathbf{r}}\,(t)\big)\,\|\vec{\mathbf{r}}\,'(t)\|\,dt$$

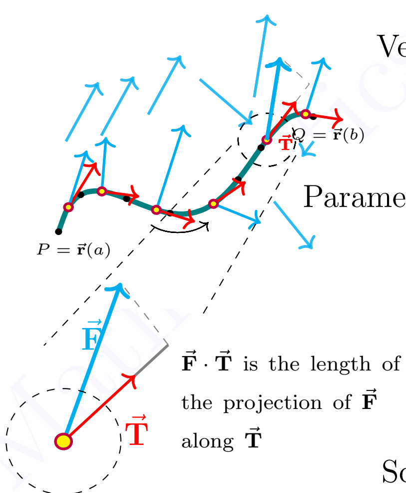
If $\mathcal{C}$ is a smooth curve in $\mathbb{R}^2$ parametrized by a function $\vec{\mathbf{r}}(t)$, and f is continuous on $\mathcal{C}$, then

$$\underbrace{\int_{\mathcal{C}} f(x,y)\,ds}_{\text{Notation}} = \underbrace{\int_a^b f\big(\vec{\mathbf{r}}\,(t)\big)\,\|\vec{\mathbf{r}}\,'(t)\|\,dt}_{\text{Formula}}.$$

The symbol $ds = \|\vec{\mathbf{r}}\,'(t)\|\,dt$ is called the **arc length element**. It represents a little bit of the arc length of the curve.

Vector Line Integral:

Let $\vec{\mathbf{F}}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z)\rangle$ be a vector field and $\mathcal{C}$ a curve parametrized by $\vec{\mathbf{r}}\,(t) = \langle x(t), y(t), z(t)\rangle$ for $[a,b]$.



Vector Differential Form: $$\underbrace{\int_{\mathcal{C}} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}}_{\text{Notation}} = \underbrace{\int_{\mathcal{C}} \vec{\mathbf{F}} \cdot \vec{\mathbf{T}}\,ds}_{\text{Physical Property}}$$

Parametric Vector Evaluation: $$\underbrace{\int_a^b \vec{\mathbf{F}}\big(x(t), y(t), z(t)\big) \cdot \vec{\mathbf{r}}\,'(t)\,dt}_{\text{A formula}}$$

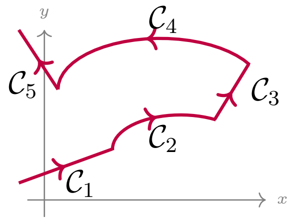
$$= \underbrace{\int_a^b \vec{\mathbf{F}}\big(\vec{\mathbf{r}}(t)\big) \cdot \vec{\mathbf{r}}\,'(t)\,dt}_{\text{Another format of the same formula}}$$

Scalar Differential Form: $$\int_{\mathcal{C}} P\,dx + Q\,dy + R\,dz$$

Parametric Scalar Evaluation:

$$\int_a^b (P(x(t),y(t),z(t))x'(t) + Q(x(t),y(t),z(t))y'(t) + R(x(t),y(t),z(t))z'(t))dt$$

Piecewise-Smooth Curves



$\mathcal{C}$ is **piecewise-smooth** if $\mathcal{C}$ is the union of a finite number of smooth curves $\mathcal{C}_1$, $\mathcal{C}_2$, ..., $\mathcal{C}_n$.

In that case,

$$\int_{\mathcal{C}} f\,ds = \int_{\mathcal{C}_1} f\,ds + \int_{\mathcal{C}_2} f\,ds + \cdots + \int_{\mathcal{C}_n} f\,ds$$

and

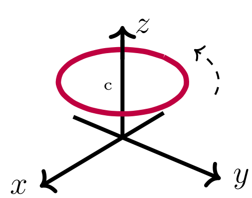
$$\int_{\mathcal{C}} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{\mathcal{C}_1} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}_1 + \int_{\mathcal{C}_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} + \cdots + \int_{\mathcal{C}_n} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}_n$$

Group Work Portion of Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. Evaluate $\int_{\mathcal{C}} \langle 0, xz, 2y \rangle \cdot d\vec{\mathbf{r}}$ where $\mathcal{C}$ is the circle of radius 2 in plane $z = 3$ and center at $(0, 0, 3)$ in the direction shown.



2. **Background Story:** Please read the instructions in the footnote.

   Questions: A 170 pound person carries a 25 pound can of paint up a helical staircase that encircles a tower with a radius of 28 feet. The tower is 180 feet high and makes exactly three complete revolutions. If there is a hole in the can of paint and 12 pounds of paint leaks steadily out of the can during the person's ascent, how much work is done by the person against gravity in climbing the stairs?

   [1]

[1] *Steps:*

- *Parametrize the staircase using the radius, the height and the number of rotations. (Refer to Section 13.3 lecture notes for an example.)*
- *Compute the vector field of downward gravitational force (Parallel to z-axis) at any point on the staircase. Note that the weight is decreasing over time in a linear fashion.*
- *Note that work is computed using the* **vector** *line integral.*

GroupWork Rubrics:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

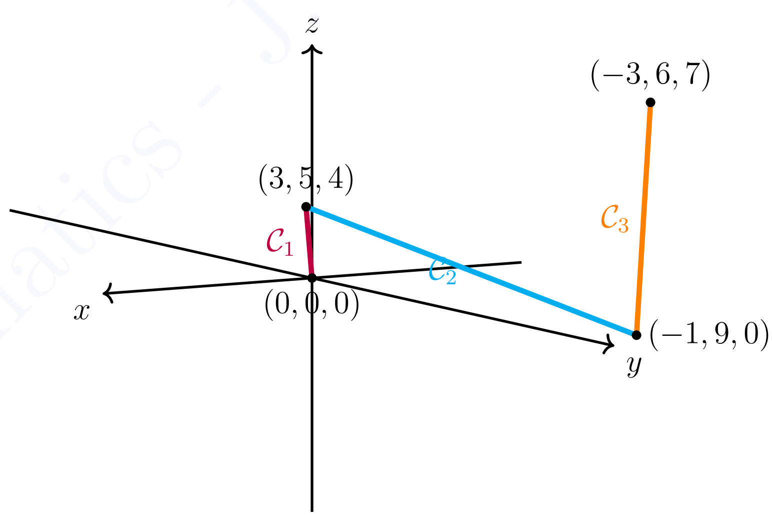
3. *(3.5 points)* **Background Story:** Follow the instructions in the footnote. Specially learn how to parametrize a line segment.

   Questions:

   Define $\mathcal{C} = \mathcal{C}_1 + \mathcal{C}_2 + \mathcal{C}_3$ where $\mathcal{C}_1$ is the line segment from $(0,0,0)$ to $(3,5,4)$, $\mathcal{C}_2$ is the line segment from $(3,5,4)$ to $(-1,9,0)$, and $\mathcal{C}_3$ is the line segment from $(-1,9,0)$ to $(-3,6,7)$. Set up the following **scalar line integral** as a single variable integral. Do not evaluate.

$$\int_{\mathcal{C}} xyz^2 \, ds$$

[2]



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[2]

- A very useful parameterization of a line segment from point (a,b,c) to (d,e,f) is $\vec{\mathbf{r}}(t) = \langle a,b,c\rangle + t\,\langle d-a, e-b, f-c\rangle$ where $0 \le t \le 1$.
- Parameterize each line segment $\mathcal{C}_1$, $\mathcal{C}_1$, and $\mathcal{C}_3$, with $\vec{\mathbf{r}}_1(t)$, $\vec{\mathbf{r}}_2(t)$ and $\vec{\mathbf{r}}_3(t)$; where each $\vec{\mathbf{r}}_i$ is the parametrization obtained by the above method.
- Then compute $\|\vec{\mathbf{r}}_i'(t)\|$ and compute $\int_{\mathcal{C}_i} f(x,y,z)ds = \int_0^1 f(\vec{\mathbf{r}}_i(t))\,\|\vec{\mathbf{r}}_i'(t)\|\,dt$ for each $i = 1,2,3$.
- Compute $\int_{\mathcal{C}} f(x,y,z)ds = \int_{\mathcal{C}_1} f(x,y,z)ds + \int_{\mathcal{C}_2} f(x,y,z)ds + \int_{\mathcal{C}_3} f(x,y,z)ds$.