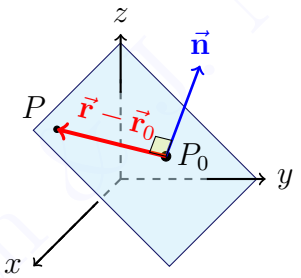
Week 2-Lab 1 or 2: Worksheet 2: Vector Review

They asked: "How do I solve problems that I have never encountered before?" I answered: "Learn the related material from lecture and lab; read the problem and identify the objective; then identify intermediate objectives and use all your math knowledge, from the beginning of your time to now, logic and common sense to translate each intermediate objective to math and to solve."

Short Descriptions and Formulas

Equations for Planes

$P_0(x_0, y_0, z_0)$: point in $\mathbb{R}^3$
$\vec{\mathbf{r}}_0 = \langle x_0, y_0, z_0 \rangle$
$\vec{\mathbf{n}} = \langle n_1, n_2, n_3 \rangle$: nonzero vector



Then there is a unique plane F that passes through P_0 and is orthogonal to $\vec{\mathbf{n}}$.

Let $P(x, y, z)$ be a general point on the plane F and let $\vec{\mathbf{r}} = \langle x, y, z \rangle$.

Vector equation of F	$\vec{\mathbf{n}} \cdot (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$
Scalar equation of F	$n_1(x - x_0) + n_2(y - y_0) + n_3(z - z_0) = 0$

The vector $\vec{\mathbf{n}}$ is called a **normal vector** to F.
Any nonzero multiple of $\vec{\mathbf{n}}$ is also a normal vector to F.

Equations of a Line in 3-Space

Let L be a line in $\mathbb{R}^3$, with direction vector $\vec{\mathbf{v}} = \langle v_1, v_2, v_3 \rangle$, containing a point $P_0 = (x_0, y_0, z_0)$.

Vector form
$\vec{\mathbf{r}} - \vec{\mathbf{r}}_0 = t\vec{\mathbf{v}}$ for all t
$\vec{\mathbf{r}}(t) = \langle x_0 + tv_1,\ y_0 + tv_2,\ z_0 + tv_3 \rangle$

Parametric form $x = x_0 + tv_1,\ y = y_0 + tv_2,\ z = z_0 + tv_3$

Symmetric form $\dfrac{x - x_0}{v_1} = \dfrac{y - y_0}{v_2} = \dfrac{z - z_0}{v_3}$
(provided $v_1, v_2, v_3 \neq 0$)

The symmetric form consists of two equations on x, y, z, with no parameter.

The vector form and parametric form are more or less the same.

GroupWork Portion of the Worksheet

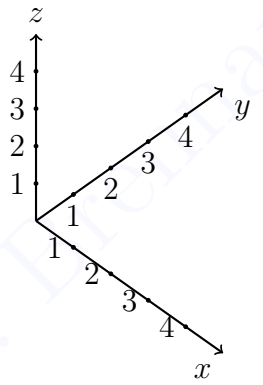
Names: _____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** Sketching in 3d is an essential component of learning vector calculus. This question is promoting that.

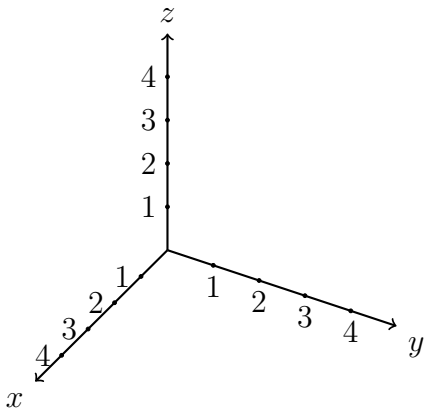
   Questions:

   (A) Sketch a triangular piece of the plane passing through points $(2, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 4)$. Then draw a normal vector to the plane. Label all items and type your name next to them.



   (B) Sketch a triangular piece of plane $x + y + z = 1$ in first quadrant. Label x, y, z intercepts of the plane. Sketch Label all items and write your name next to them.

   (C) Sketch the plane $y = 3$. Label any intercept of the plane that exist. Sketch Label all items and write your name next to them. Sketch two unit normal vectors to this plane.



2. **Background Story:** In Calculus II, we learned many methods of integration. The general rule of thumb to follow the lecture pace in Calculus III, Differential Equation or your physics/ engineering courses is to know some of the methods by heart. We only worry about the ones that we do in Calculus III. The next questions are good samples of what methods you need to know by heart.

   Questions: Choose a method of solving for each of the following integrals. That is, give u for a u-substitution, give u and v for integration by parts, symmetry, or the trigonometric substitution used.

   (A) $\displaystyle\int \sin(x)\cos^5(x)\,dx$

   (C) $\displaystyle\int \cos^2(x)\,dx$

   (E) $\displaystyle\int \frac{x}{1+x^2}\,dx$

   (B) $\displaystyle\int x^3\cos(x)\,dx$

   (D) $\displaystyle\int_0^{2\pi} \cos^5(x)\,dx$

   (F) $\displaystyle\int \frac{1}{25+x^2}dx$

3. **Background Story:** When talking about intersections of lines and planes, it is important to connect the concepts to systems of linear equations (Yes, as in Linear Algebra). Make the connections in the following questions.

   Questions: Consider the two planes

   $$-2x + 2y + 7z = 1 \qquad\qquad 2x + y + 2z = 5.$$

   (A) Solve a system of linear equations to find the line of intersection of the two planes.

   (B) What is a direction vector of the line of intersection of the plane?

   (C) $\vec{n}_1 = \langle -2,\ 2,\ 7 \rangle$ and $\vec{n}_2 = \langle 2,\ 1,\ 2 \rangle$ are normal to the planes. Explain why $\vec{n}_1 \times \vec{n}_2$ is parallel to the line of intersection.

   (D) Find a vector of magnitude 5 parallel to the line of intersection of the planes using the cross product.

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: _____

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

4. Solve each integral using the information from GroupWork Question 2.

(1) *(0.5 points)* $\int \sin(x)\cos^5(x)\,dx$

(2) *(0.5 points)* $\int x^3\cos(x)\,dx$

(3) *(0.5 points)* $\int \cos^2(x)\,dx$

(4) *(0.5 points)* $\int_0^{2\pi} \cos^5(x)\,dx$

(5) *(0.5 points)* $\int \frac{x}{1+x^2}\,dx$

(6) *(0.5 points)* $\int \frac{1}{25+x^2}dx$

5. A symmetric equation of the line $\vec{\mathbf{r}}(t)$ is $\dfrac{x-3}{-4} = \dfrac{y+5}{3} = \dfrac{z-1}{2}$.

(A) *(1.5 points)* Find the vector form (or parametric form) of the line $\vec{\mathbf{r}}(t)$.

(B) *(1 point)* Find the point in which the line $\vec{r}(t)$ intersects the plane $3x - y + z = -24$.

6. *(1.5 points)* Find an equation of the plane through point $P(2, -4, 2)$ that contains the line $\vec{r}(t) = \langle 2 + 3t, t, -5 - 3t \rangle$.

Math 127 and material, open letter to my students

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We also receive reports of tutors on social media platforms such as Groupme who use our previous semester material and, instead of really helping you, they just give you the solution to material. This activity is academic dishonesty. Some of these tutors are paid by small groups and they seem to help everyone in the social media. Please only get help from those tutors if they are willing to be on the same social media platform(s) as your instructors or add us to the social media platform they are using. Please don't get tricked by their gimmicks to create a divide between you and your instructors. That "divide" is how they profit with very small amount of work. We are all about helping each other out in a way that helps you learn more. So please have us involved in your problem solving process.