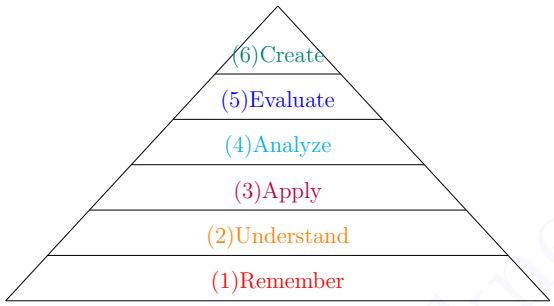
Week 3-Lab 1: Worksheet 3: Section 12.6 and 14.1

I said: "Look at this Pyramid, what you are doing in any lecture is the two lower blocks of the pyramid. When assignments are very similar to lecture problems, you are doing the third block. When assignments are different from lecture but guided (some of the worksheets) you are on the fourth and fifth blocks. The top block is only achieved when assignments are different from lecture with little guidance. You need to try all steps to really learn." " They asked: " Do you have recommendation for tutors?" I said: " Our help room, our office hours, SAC and school of engineering are free."



Learning Pyramid

Short Descriptions and Formulas

Functions of Two Variables

A **function** $f : \mathbb{R}^2 \to \mathbb{R}$ **of two variables** is a rule that assigns to each ordered pair (x, y) in a set D a unique number denoted by $f(x, y)$.

The set D is called the **domain** of f.

The **range** of f is the set of output values of f for points in D.

$$\text{Range}(f) = \{f(x, y) \,|\, (x, y) \in D\}$$

The domain is a subset of $\mathbb{R}^2$, and the range is a subset of $\mathbb{R}$.

Often we write $z = f(x, y)$ for pairs (x, y) in D.

x and y are the **independent** variables of f.

z is the **dependent** variable of f.

Cross Sections and Level Curves

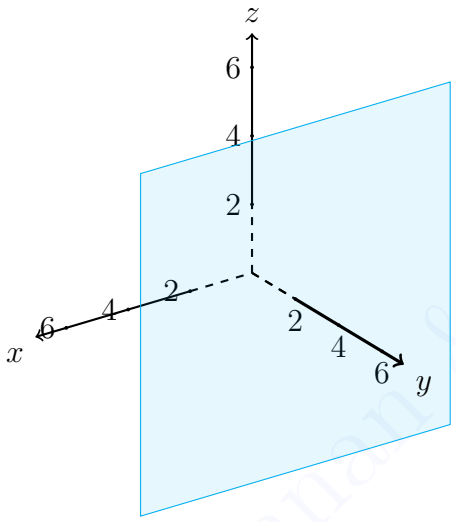
The cross sections of a surface are the intersections of a surface with planes. For **Quadric** surfaces in this course, we look at the planes $x = k$, $y = k$ or $z = k$.

Level Curves are the cross sections of the graph of two variables with plane $z = k$.

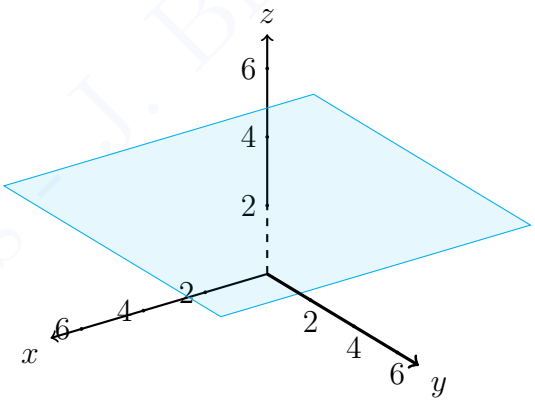
1. **Background Story:** Finding a cross section of a surface means finding the intersection of a surface with a plane. These intersections, if they exist, could be point(s), lines and curves in those planes.[1] In the next question, practice graphing curves in planes in 3d.

   Questions:

   (A) Graph cross section of $x^2 + z^2 = y^2$ with plane $y = 2$. That is, $x^2 + z^2 = 4$



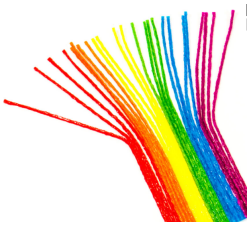
   (B) Graph cross section of $x^2 - y^2 = z$ with plane $z = 2$. That is, $x^2 - y^2 = 2$. [2]



[1] Sometimes, they are regions in the plane.

[2] Video and geogebra: https://www.geogebra.org/m/brayrbmp

2. **Background Story:** Graphing the Quadric surfaces is very much computing the cross sections, making a shape for each of the cross sections with Wikki Stix and then putting all Wikki Stix together.



Questions:

(A) Find the following cross sections for each of these surfaces.

i. $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 + 0.25\,z^2 = 1$

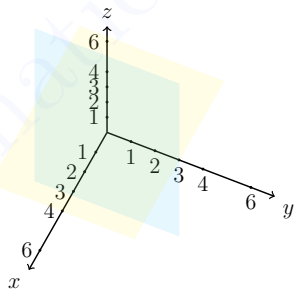
$x = 0$:	$y = 0$:
$x = 3$:	$y = 1$:
$x = -1$:	$y = -1$:
$x = 4$:	$y = 4$:
$x = 6$:	$y = -4$:

ii. $\left(\frac{x}{4}\right)^2 + \left(\frac{z}{3}\right)^2 - 0.5y = 0$

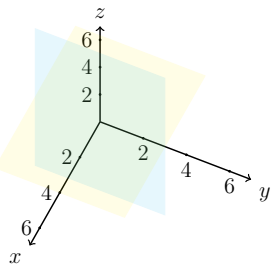
$x = 0$:	$y = 0$:
$x = 1$:	$y = 1$:
$x = -1$:	$y = -1$:
$x = 4$:	$y = 4$:
$x = 6$:	$y = -4$:

(B) Graph and label the cross sections in 3d if they exists.

(i)



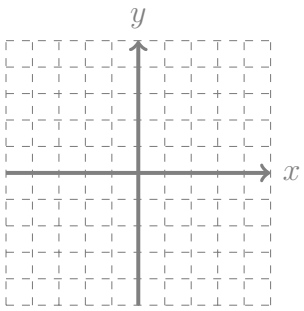
(ii)



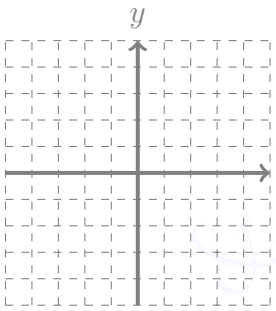
3. **Background Story:** The domain of a function of one variable is a subset of $\mathbb{R}$. The domain of function of two Variable is a subset of $\mathbb{R}^2$. To find the domain, you use the restrictions enforced by root functions, quotient functions and log functions to find a region in $\mathbb{R}^2$. Remember if you are solving inequalities switch the inequality sign to equal to find the curve(s) of boundary and then divide $\mathbb{R}^2$ into two regions using the resulting curve. Last use test point to find the solution to inequality.

   Questions:

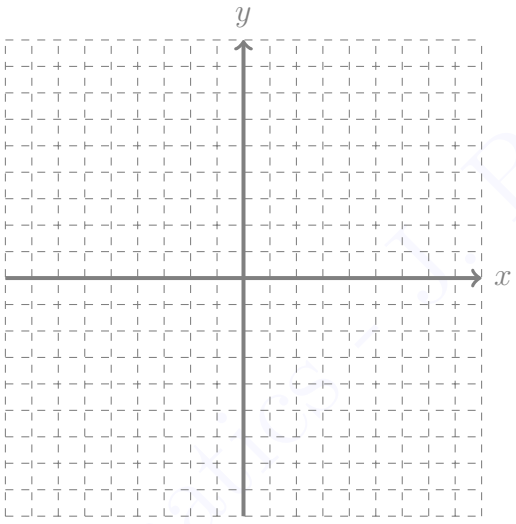
   (A) Shade the domain of $z = \sqrt{x^2 + y^2 - 4}$.

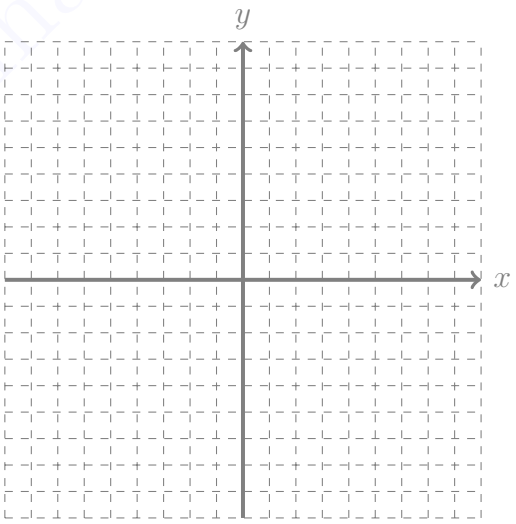
   (B) Shade the domain of $z = \sqrt{4 - (x^2 + y^2)}$.

   (C) Shade the domain of $z = \sqrt{x - y - 4}$.

   (D) Shade the domain of $z = \sqrt{4 - (x - y)}$.

4. **Background Story:** Discuss multivariable functions.

   Questions:

   (A) In a few sentences explain what property a surface can have to make it a graph of $z = f(x, y)$.

   (B) Give an example of a multivariable function in your field of study.

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of Worksheet

Name: _____

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

5. *(3.5 points)* Identify and sketch the surface.

(A) $\left(\frac{y}{3}\right)^2 - \left(\frac{x}{4}\right)^2 - 2z^2 = 1$

Identify: _____

Sketch:

(B) $\left(\frac{y}{3}\right)^2 - \left(\frac{x}{4}\right)^2 - 2z^2 = 0$

Identify: _____

Sketch:

(C) $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 + 0.25\,z^2 = 1$

Identify: _____

Sketch:

(D) $\left(\frac{y}{2}\right)^2 + \left(\frac{z}{3}\right)^2 - 0.5x = 0$

Identify: _____

Sketch: