Week 3-Lab 2: Worksheet 4: Sections 11.3 and 12.7

They said: "I went to the help room and they asked more questions than I did." I said: "They are helping you identify the problems' main goal and intermediate objectives."

Short Descriptions and Formulas

Conversion Formulas

Conversion from polar to Cartesian coordinates:

$$x = r\cos(\theta) \qquad y = r\sin(\theta)$$

Conversion from Cartesian to polar coordinates:

$$r^2 = x^2 + y^2 \qquad \tan(\theta) = \frac{y}{x}$$

Note: Diagrams for the following are part of Problem 1 of groupwork.

Cylindrical-Cartesian Conversion

Conversion from cylindrical coordinates to Cartesian coordinates:

$$x = r\cos(\theta) \qquad y = r\sin(\theta) \qquad z = z$$

Conversion from Cartesian coordinates to cylindrical coordinates:

$$r^2 = x^2 + y^2 \qquad \tan(\theta) = \frac{y}{x} \qquad z = z$$

Spherical-Cartesian Conversion

Conversion from spherical to Cartesian:

$$x = \rho\sin(\phi)\cos(\theta) \qquad y = \rho\sin(\phi)\sin(\theta) \qquad z = \rho\cos(\phi)$$

Conversion from Cartesian to spherical:

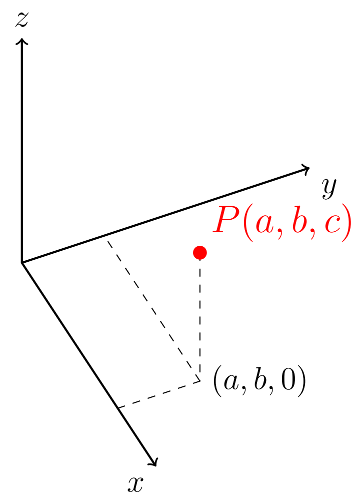
$$\rho = \sqrt{x^2 + y^2 + z^2} \qquad \tan(\theta) = \frac{y}{x} \qquad \cos(\phi) = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

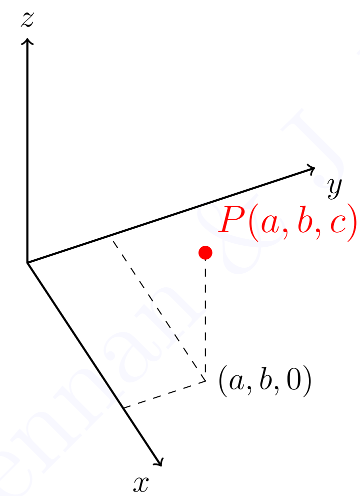
1. **Background Story:** One of the significance of spherical and cylindrical is that they symmetry of them is what is needed in many fields such as electromagnetism. In the following questions, we try to understand some of the structures using the app and sketching. Then we use the formulas and this knowledge to convert in the individual portion of the work.

   Questions:

   (A) Discuss and illustrate cylindrical conversion (r, θ, z) for point P. (Draw the angle θ and line segment r.)

   (B) Discuss and illustrate spherical conversion (ρ, ϕ, θ) for point P. (Draw the angles ϕ, θ, r and line segment ρ.)





   (C) Open the following Geogebra Sheets on your computer, share the screen and discuss the following Geogebra Sheets. Describe the cylindrical and spherical coordinates on them.

      (i) Discuss this **solid** in cylindrical coordinates: https://www.geogebra.org/m/pg2ctatn

      (ii) Discuss the spherical coordinate conversion: https://www.geogebra.org/m/wmhtgy6r

      (iii) Discuss this **surface** in spherical coordinates: https://www.geogebra.org/m/vpna3c8x

      (iv) Discuss the Question 2[1] by moving sliders a, b and c in this worksheet: https://www.geogebra.org/m/murrfwyz

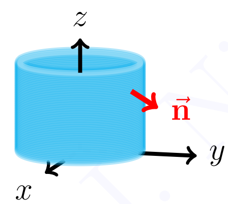
[1] You are encouraged to complete the individual question in class as long as all of you participate.

2. **Background Story:** In your Physics and Electromagnetism classes, you will see that basis other than Cartesian Basis is used. The next set of questions prepares you for that.
[2]

Questions:

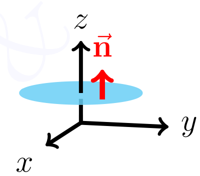
In each of the following cases $\vec{\mathbf{n}}$ is a unit vector at $\langle x,\, y,\, z\rangle$. Compute it then convert the vector to **cylindrical** $r,\ \theta,\ z$ or **spherical** $\rho,\ \theta,\ \phi$. For example, $\vec{\mathbf{n}}$ is a unit vector in direction of z-axis so $\vec{\mathbf{n}} = \langle 0, 0, 1\rangle$ and in this case no Conversion is needed.

(A) $\vec{\mathbf{n}}$ is the **unit vector** in direction $\langle x, y, 0\rangle$ and then convert x, y, z to **cylindrical** $r,\ \theta,\ z$.



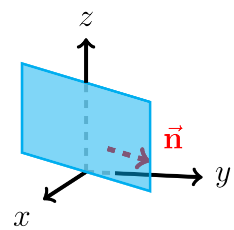
$\vec{\mathbf{n}} =$

(B) $\vec{\mathbf{n}}$ is the **unit vector** in direction of z-axis and then convert x, y, z to **cylindrical** $r,\ \theta,\ z$.



$\vec{\mathbf{n}} =$

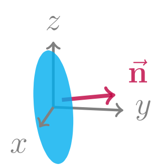
(C) $\vec{\mathbf{n}}$ is the **unit vector** in direction orthogonal to plane containing z-axis[3] in terms of **cylindrical** $r,\ \theta,\ z$.



$\vec{\mathbf{n}} =$

(D) $\vec{\mathbf{n}}$ is the **unit vector** in direction $\langle x, y, z\rangle$ and then convert x, y, z to **spherical** $\rho,\ \theta,\ \phi$.
$\vec{\mathbf{n}} =$

(E) $\vec{\mathbf{n}}$ is the **unit vector** in direction orthogonal to plane $\theta =$constant and then convert x, y, z to **spherical** $\rho,\ \theta,\ \phi$.



$\vec{\mathbf{n}} =$

[2] Normal Vectors related to Cylindrical: https://www.geogebra.org/m/z32aeh6w
Normal vectors related to Spherical: https://www.geogebra.org/m/cfybupr4

[3] That is the plane $\theta =$constant. Note that, in electromagnetism and other area of physics, we compute unit normal vector to a rotating rectangular regions or rotating disks often. So this Part and Part (E) of this problem is helping you find that. The vectors are going to be in terms of θ only. Find them however you can.

3. **Background Story:** One of the major reasons to learn spherical and cylindrical coordinates is to compute 3d integrals in Chapters 15 and 16. Those are the integrals with a high volume[4] of applications in physics, statistics, electromagnetism and other fields. One skill that you will need is to efficiently express a solid in terms of inequalities in spherical and cylindrical coordinates. The following problems are meant to shadow that. Please note that we will find inequalities with certain properties in Chapter 15 and these problems don't cover all of the steps needed in Chapter 15.

Questions:

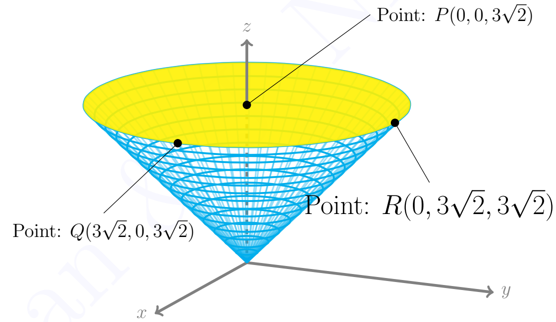
(A) Consider the solid entrapped between plane $z = 4$ and upper nape of the cone $z^2 = x^2 + y^2$.

(i) Find a point on the solid that is furthest from the origin and one that is closest.

(ii) What is the minimum and the maximum value of ρ in this piece of upper nappe of a cone? (Enter two numbers.)
$\boxed{} \leq \rho \leq \boxed{}$

(iii) Find the point in the solid that has the largest ϕ.

(iv) What is the minimum and the maximum value of ϕ for any points on this solid? (Enter two numbers.) $\boxed{} \leq \phi \leq \boxed{}$



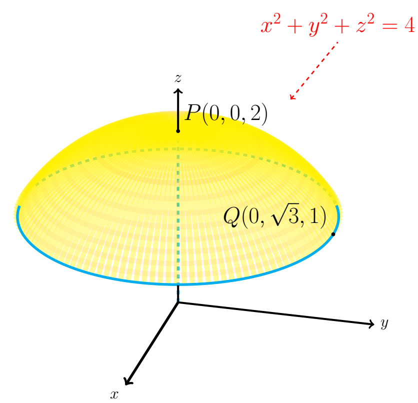
(B) Consider the solid entrapped between the sphere $x^2 + y^2 + z^2 = 4$ and $z = 1$.

(i) Find a point on the solid that is furthest from the origin.

(ii) What is the minimum and the maximum value of ρ on any point of the solid? (Enter two numbers.)
$\boxed{} \leq \rho \leq \boxed{}$

(iii) Find the point in the solid that has the largest ϕ.

(iv) What is the minimum and the maximum value of ϕ for any point on the solid? (Enter two numbers.) $\boxed{} \leq \phi \leq \boxed{}$



[4] Playing with words! Aren't we?

4. **Background Story:** Let's have a blast from the past! What does a surface of rotation look like? What if plane containing the area also rotates with the area?

Questions:

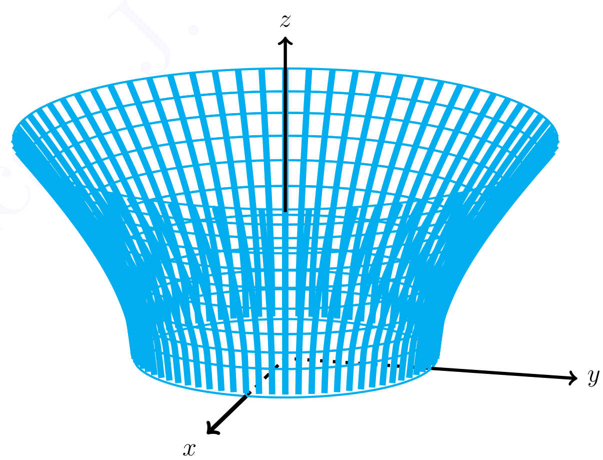
Consider the surface of $z = \sqrt{x^2 + y^2 - 1}$.

(a) Convert the Cartesian surface to Cylindrical coordinates.

(b) Algebraically find the cross section of the surface with the plane $\theta = k$.

(c) Does answer to Part (B) change when k changes?

(d) Draw the curve of cross section of two planes $\theta = k_1$ and $\theta = k_2$ with the hyperbolic paraboloid for two values $k_1 \neq k_2$ of your choosing.



(e) Discuss with your friends what you think is the surface of rotation in Calculus III.

GroupWork Rubrics:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

5. **Background Story:** A major goal of this section is to familiarize you with conversion of equations of surfaces to cylindrical or spherical coordinates. This sets up a good foundation when we compute volume with spherical or cylindrical symmetry.

   Questions:

   (A) *(1 point)* Find a spherical coordinate equation for $x^2 + y^2 + (z-3)^2 = 9$.

   (B) *(1.5 points)* Find a spherical coordinate equation for $z = \sqrt{x^2 + y^2}$.

6. *(1 point)* Match each Item (i)-(iv) to an Item (A)-(J).

(A) A circle centered at origin with radius 2

(B) A Sphere centered at origin with radius 2

(C) A Sphere centered at $(0, 0, 1)$ with radius 2

(D) The xy-plane

(E) Half xy-plane

(F) Half line $x = y = 0$ and $z \leq 0$

(G) A half plane perpendicular to xy-plane

(H) A cone

(I) Upper nape of a cone

(J) Lower nape of a cone

(i) $\rho = 2$

(ii) $\phi = \pi$

(iii) $\phi = \dfrac{2\pi}{3}$

(iv) $\theta = \dfrac{\pi}{3}$