Week 1-Lab 1 or 2: Worksheet 1: Vector Review

They asked: "How do I study for this course?"
I said: "Learn the material every day."
They asked: "How do I learn the material every day?"
I replied: "Read the appropriate section of the book or notes before lecture, download the pre-lecture notes, follow the lectures without distraction of your phones and other devices, work on worksheets before going to the labs, continue working on your worksheets in the labs to learn the material hands-on, and do the achieve assignments to practice the skills you have learned, take the diagnostic quizzes related to each In-class quiz and exam; retake the diagnostic quizzes if the first score is not desirable. Make sure to get help in our help room or at one of the free tutoring places on campus if you have any questions. Aforementioned are the most important part of learning. Then do all problems on review on your own the week before the exam to study."

Short Descriptions and Formulas

Arc Length Formula

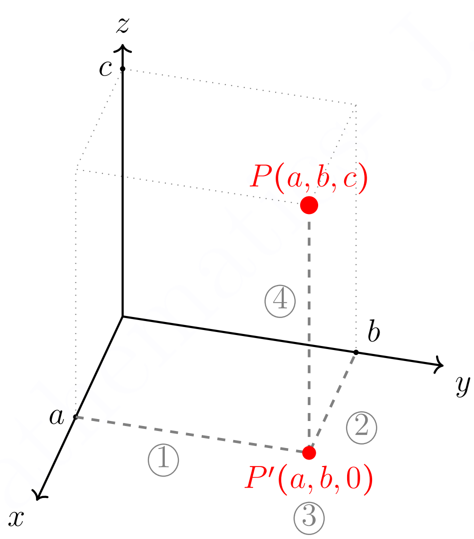
The arc length of the curve parametrized by $\vec{\mathbf{r}}(t)$ for $a \le t \le b$ is

$$\int_a^b \|\vec{\mathbf{r}}\,'(t)\|\,dt.$$

The arc length of graph of the function $y = f(x)$ for $a \le x \le b$ is

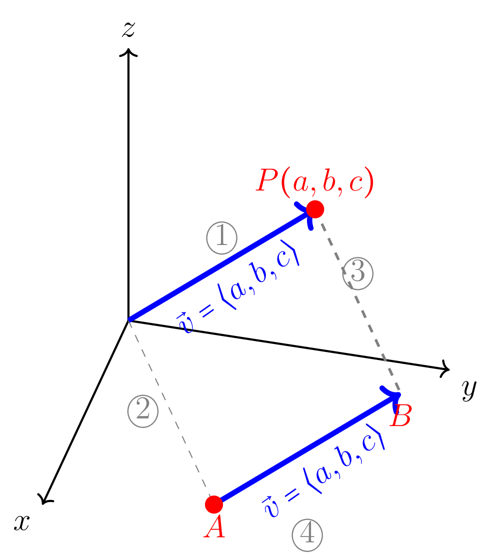
$$\int_a^b \sqrt{1 + (y'(x))^2}\,dx.$$

How to draw a point in 3d:



To draw point $P(a,b,c)$: ① Draw a line parallel to y-axis from point $(a,0,0)$. ② Draw a line parallel to x-axis from point $(0,b,0)$. ③ Name the intersection of the two lines P'. ④ Draw a line parallel to z-axis from point $(a,b,0)$ and mark c units to get to $P(a,b,c)$.

How to draw a vector in 3D:



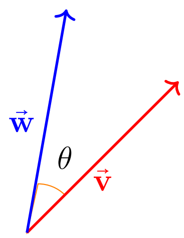
To draw the vector $\vec{v} = \langle a,b,c\rangle$ at an initial point A: ① First draw point (a,b,c) and connect $(0,0,0)$ to (a,b,c). ② Connect $(0,0,0)$ and A. ③ Draw a line segment parallel and the same length as to line segment in ② from $P(a,b,c)$. Denote the new end point B. ④ Draw the vector $\vec{v}$ from A to B.

Dot Product

The **dot product** of two vectors $\vec{\mathbf{v}} = \langle a_1, b_1, c_1 \rangle$ and $\vec{\mathbf{w}} = \langle a_2, b_2, c_2 \rangle$ is the scalar

$$\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = \|\vec{\mathbf{v}}\| \, \|\vec{\mathbf{w}}\| \, \cos(\theta)$$

where θ is the angle between the vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$.



- If $0 \le \theta < \frac{\pi}{2}$ then $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} > 0$.
- If $\theta = \frac{\pi}{2}$ then $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = 0$.
- If $\frac{\pi}{2} < \theta \le \pi$ then $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} < 0$.
- The angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ is $\arccos\left(\dfrac{\vec{\mathbf{v}} \cdot \vec{\mathbf{w}}}{\|\vec{\mathbf{v}}\| \, \|\vec{\mathbf{w}}\|}\right)$.

The Cross Product

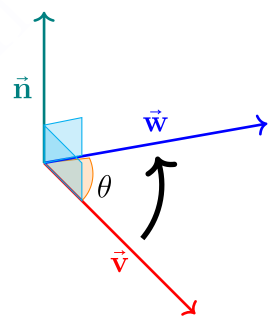
The **cross product** of vectors $\vec{\mathbf{v}}, \vec{\mathbf{w}}$ in $\mathbb{R}^3$ is the vector

$$\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \big(\|\vec{\mathbf{v}}\| \, \|\vec{\mathbf{w}}\| \, \sin(\theta)\big) \, \vec{\mathbf{n}}$$

where:

(i) θ is the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$;

(ii) $\vec{\mathbf{n}}$ is the unit vector perpendicular to both $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$, given by the Right-Hand Rule.



Cross Product Formula: $\quad \vec{\mathbf{v}} \times \vec{\mathbf{w}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

The Formula for the Dot Product

Formula in $\mathbb{R}^2$: $\quad \vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = a_1 a_2 + b_1 b_2$

Formula in $\mathbb{R}^3$: $\quad \vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = a_1 a_2 + b_1 b_2 + c_1 c_2$

Dot and Cross Products

(1) The **dot product**, which takes two vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ (either both in $\mathbb{R}^2$ or both in $\mathbb{R}^3$) and produces a *scalar* $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}}$.

(2) The **cross product**, which takes two vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ (both in $\mathbb{R}^3$) and produces a *vector* $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$.

It is very important to understand the **geometry** behind the dot and cross product, not just their formulas.

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Ice Breaker Background Story:** Take a quick minute or two, turn around and talk to your immediate neighbors; find friends and group mate for the rest of the semester. We will ask you these questions on your introduction survey as well. Please fill out that survey so we can know you better.

   (1) Your preferred name,

   (2) What your major is,

   (3) Where you're from,

   (4) Who you are,

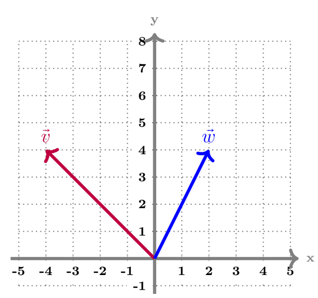
   (5) Something we would not know just by looking at you.

   Note: The plan for Monday/Wednesday labs is to complete at least Questions 2-5; work on 6-8 on their own and choose at lest one to do on Monday. Tuesday/Thursday lab should complete 2-5 on Tuesday and get the rest done on Thursday. If your group doesn't get to all of the problems, please work on the rest on your own.

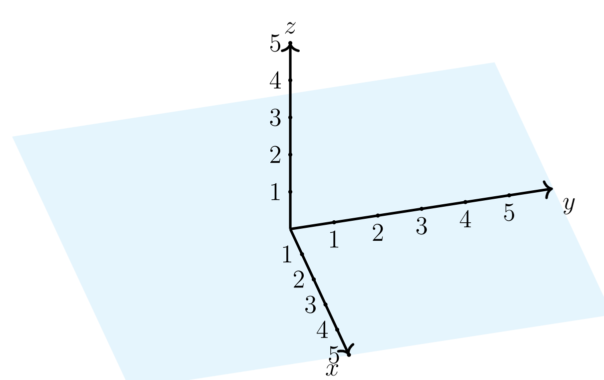
2. **Background Story:** 3d designs are integrated parts of many fields of engineering. Understanding how to sketch points and lines in 3d is the first step in understanding how to sketch in 3d.

   Questions:

   (A) sketch the following vectors $\vec{i} = \vec{v} + \vec{w}$, $\vec{ii} = \vec{v} - \vec{w}$ and $\vec{iii} = -1.5\vec{v}$ using the parallelogram, triangle or other geometric properties. Label all items.

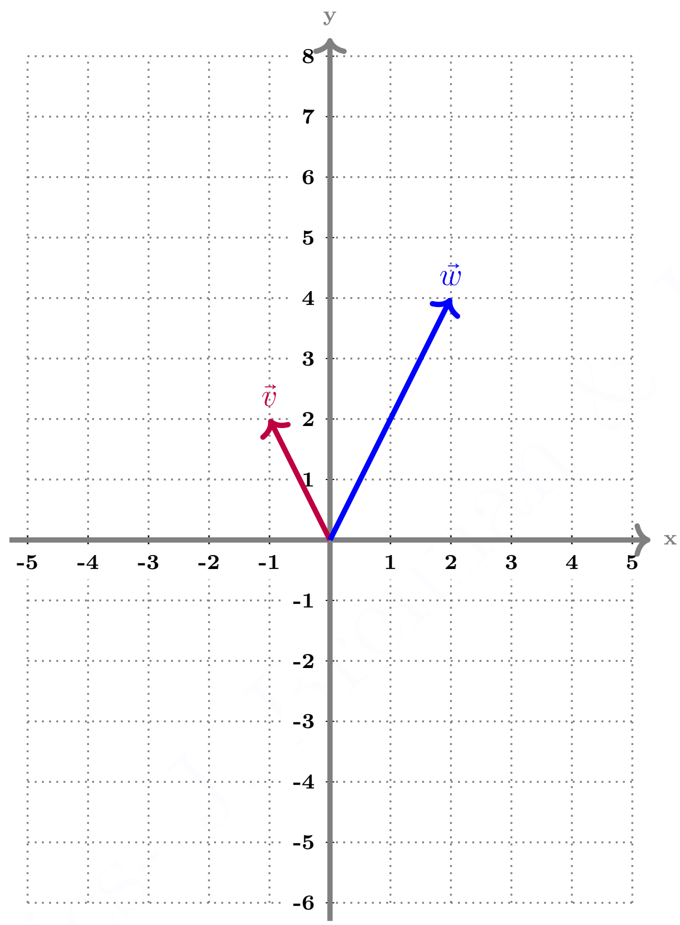


   (B) Sketch points $A = (-4, 2, 0)$ and $B = (-4, 2, 4)$, vector $\vec{v} = \langle -4, 2, 4 \rangle$ and a line ("l") through two points A and B . Label all items.



3. **Background Story:** The beauty of using the unit Cartesian basis $\vec{\mathbf{i}}$ and $\vec{\mathbf{j}}$ is that we can compute the vector sum, difference and scalar multiplication without sketching. In the next question, sketch and compute using Cartesian coordinates.

   Questions: Evaluate the following operations where the $\vec{v}$ and $\vec{w}$ are shown in the figure. Use the correct notation for the vectors, $\langle\ ,\ \rangle$.



(i) $\vec{v} + \vec{w} =$

(ii) $\vec{v} - \vec{w} =$

(iii) $-4\vec{v} =$

(iv) $3\vec{v} =$

(v) $\vec{v} \cdot \vec{w} =$

(vi) $\vec{v} + 2\langle 1, 0\rangle =$

4. **Background Story:** Knowing that have right angles in 3d is important. One application of dot product is in computing the angle between vectors given in Cartesian coordinates.

Questions:

(A) Is $\langle 0, 1, 5\rangle$ orthogonal to $\langle -10, 1, 2\rangle$? How do you know? If you said no, compute the angle between them.

(B) Is $\langle 0, 1, 5\rangle$ orthogonal to $\langle 1, -10, 2\rangle$? How do you know? If you said no, compute the angle between them.

(C) Find two unit vectors, in terms of t, that are orthogonal to $\vec{v} = \langle \cos(t), \sin(t)\rangle$.

(D) What is a directional vector of line $\langle 2 + 5t, 2, 7 + t\rangle$?

5. **Background Story:** In Calculus II, we learned many methods of integration. The general rule of thumb to follow the lecture pace in Calculus III, Differential Equation or your physics/ engineering courses is to know some of the methods by heart. We only worry about the ones that we do in Calculus III. The next questions are good samples of what methods you need to know by heart.

Questions: Choose a method of solving for each of the following integrals. That is, give u for a u-substitution, give u and v for integration by parts, symmetry, or the trigonometric substitution used.

(A) $\int \sin(x)\cos^3(x)\,dx$

(B) $\int x^3\cos(x)\,dx$

(C) $\int \cos^2(x)\,dx$

(D) $\int_0^{2\pi} \cos^3(x)\,dx$

(E) $\int \frac{x}{1+x^2}\,dx$

(F) $\int \frac{1}{9+x^2}dx$

[1]

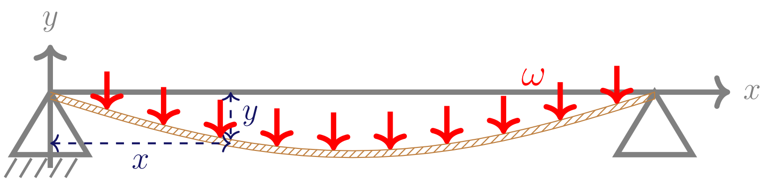
[1] Videos: https://youtu.be/repkbIFD9aM and https://youtu.be/cYAm8ZQ5Q94

6. **Deflection of a Simply supported bridge, Supported on both Ends, with a Uniformly Distributed Load**

As a load is uniformly distributed on a simply supported bridge of length $L = 20\,ft$; the vertical displacement of the beam from the horizontal, y in ft, at a horizontal distance $x\,ft$ from the left support is best approximated by the function

$$y = \frac{-x}{244,488}\left(8000 - 40\,x^2 + x^3\right)$$

Where

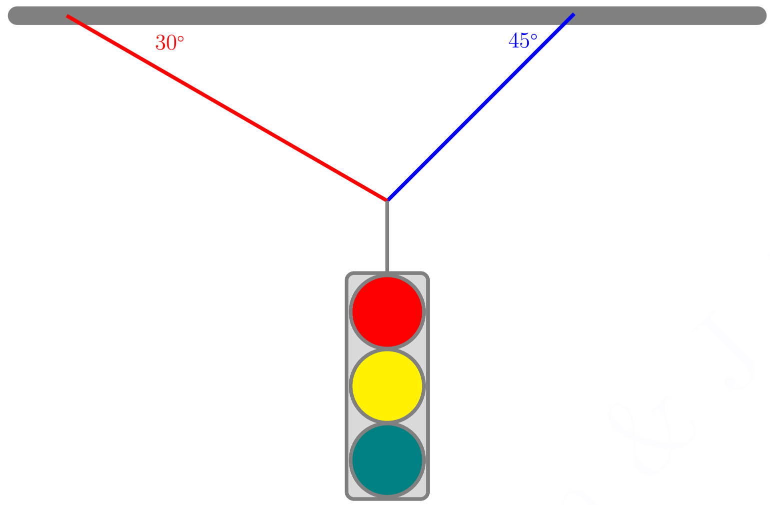


An ant travels along the bridge starting at the left support for $10\,ft$; what is the horizontal distance of the ant?

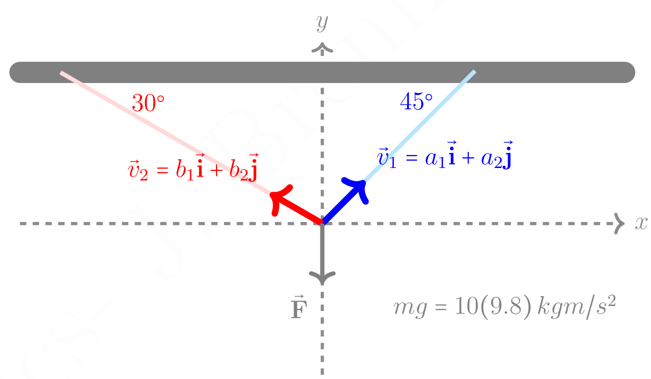
(You can use wolframalpha to get a good approximation. Change the upper bound here:
https://www.wolframalpha.com/input?i=int_0%5E9.99+sqrt%7B%28%288000-120t%5E2%2B4t%5E3%29%2F244488%29%5E2%2B1%7D+dt.)

7. **Background Story:** In Physics and many fields of engineering, we find tensions in a 3d space. The solution to 3d problem is a generalization of the 2d problem.

Questions: A 10 kg traffic light is hanging from a rod by two ropes and is in equilibrium position as shown in this picture. We are interested in computing the tension on each rope.



(A) Represent the force of gravity as a vector in $\mathcal{R}^2$, denoted by $\vec{\mathbf{F}}$.



(B) Use trigonometry to relate a_1 and a_2. (Give an equation) Use the trigonometry to relate b_1 and b_2. (Give another equation.)

(C) "The traffic light is in equilibrium" means the net forces is zero ($\vec{v}_1 + \vec{v}_2 + \vec{\mathbf{F}} = \vec{0}$). What does it mean in terms of x-components of the vectors? (Give an equation.) What does it mean in terms of y-components of the vectors? (Give another equation.)

(D) Use the four equations and solve for a_1, a_2, b_1, and b_2.

(E) Rewrite $\vec{v}_1$ and $\vec{v}_2$ with numeric values you found in Part (D).

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

8. **Background Story:** Cross products are good tools in computing a vector orthogonal to two non-parallel vectors. This fact will be helpful in finding an equation of a plane.

   Questions: Consider the two vectors $\vec{u} = \vec{j} + 3\vec{k}$ and $\vec{v} = 2\vec{i} - 4\vec{j} + \vec{k}$.

   (A) *(0.75 points) Compute $\vec{u} \times \vec{v}$.*

   (B) (0.5 points) How many unit vectors are orthogonal to $\vec{u}$? How about $\vec{v}$?

   (C) (0.75 points) Find two unit vectors that are orthogonal to both $\vec{u}$ and $\vec{v}$.

9. Solve each integral.

(1) *(1 point)* $\int \sin(x)\cos^3(x)\,dx$

(4) *(0.75 points)* $\int_0^{2\pi} \cos^3(x)\,dx$

(2) *(1 point)* $\int x^3\cos(x)\,dx$

(5) *(0.75 points)* $\int \frac{x}{1+x^2}\,dx$

(3) *(0.75 points)* $\int \cos^2(x)\,dx$

(6) *(0.75 points)* $\int \frac{1}{9+x^2}dx$

Math 127 and material, open letter to my students

Dear friends of Math 127,

We often receive reports of different websites that sell our material without our permission. They also pay small fees or allow free access to our students to obtain these material. Please note that this activity is illegal and is a form of intellectual property theft of your instructors. Please don't be tricked by these websites. We spend long hours <u>outside our working hours</u> to create these material so our students can use them for **free**.

We also receive reports of tutors on social media platforms such as Groupme who use our previous semester material and, instead of really helping you, they just give you the solution to material. This activity is academic dishonesty. Some of these tutors are paid by small groups and they seem to help everyone in a social media platform. Please only get help from tutors if they are on your instructors' social media. Please don't get tricked by their gimmicks to create a divide between you and your instructors. That "divide" is how they profit with very small amount of work. We are all about helping each other out in a way that helps you learn more. So please have us involved in your problem solving process.