Week 7-Lab 2: Worksheet 10 Sections 14.8

She said: " I was solving the questions correctly for my group but every time, they preferred his answers over mine. He was giving the exact same answers." I said: " I am so sorry!" I asked my daughter: "What did you do when this happened to you?" She said: "I had an ally. He said that I did a better job answering the questions than he did. That reduced the pain." I am asking you: " Be an ally!"

The Method of Lagrange Multipliers

To find the extrema of $z = f(x, y)$ subject to the constraint $g(x, y) = k$,

(1) Find all values a, b, and λ such that $g(a, b) = k$ and

$$\nabla f(a, b) = \lambda \nabla g(a, b)$$

- * There are a total of three equations and three unknowns.
- * Often, the best way to solve this system is to start by eliminating λ.

(2) Calculate the values of f at all points (a, b) found in step (1) and the end points.

(3) The largest of these is the absolute maximum value and the smallest is the absolute minimum value of f constrained by $g(x, y) = k$ if the constraint is closed and bounded.

The same method works for functions of three or more variables.

Lagrange Multipliers with Two Constraints

To find the extrema of $f(x, y, z)$ subject to two constraints $g(x, y, z) = k$ and $h(x, y, z) = m$:

(1) Solve the system of five equations and five unknowns:

$$\begin{aligned} \nabla f(a, b, c) &= \lambda \nabla g(a, b, c) + \mu \nabla h(a, b, c) \\ g(a, b, c) &= k \\ h(a, b, c) &= m \end{aligned}$$

(2) Calculate the values of f at all points (a, b, c) found in step (1) and the end points.

(3) The largest of these is the absolute maximum value and the smallest is the absolute minimum value if the curve of the intersection is closed and bounded.

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** This problem is to help you choose a method to optimize a function on a curve or a collection of curves. It also helps you understand that you can interchange methods.

   Questions: Assume that $f(x, y)$ is a differentiable function.

   (a) Explain two methods to find extrema of f subject to constraint $x^2 + y^2 = 25$. (Hint: Use a method from each of the Sections 14.7 and 14.8.)

   (b) Explain two methods to find extrema of f subject to constraint $x + y = 25$ and $x, y \geq 0$. (Hint: Sections 14.7 and 14.8.)

   (c) Which of the two methods do you prefer in each case? How about when the constraint is a collection of multiple pieces of line segments?

2. The base of a rectangular aquarium with given constant volume V is made of slate and the sides are made of glass. If slate costs six times as much (per unit area) as glass, find the dimensions of the aquarium that minimize the cost of the materials. (Use Lagrange Multipliers.)

Video: https://youtu.be/EHRlGSC-zRo

3. **Background Story:** Sometimes we can replace an objective function with one that has less-complicated derivatives. The next question is discussing the distance function. Can we replace that with a simpler function? Investigate.

   Questions: Find the point on the plane $2x + 3y + 5z = 76$ that is nearest the origin.

   (a) What is the objective function, f, and what is the constraint, g?

   (b) Write four Lagrange multiplier equations, including the constraint.

   (c) Solve for the point. Find the distance of the point from the origin.

Video: https://youtu.be/xnzoCDJKPJQ

4. **Background Story:** The Lagrange multiplier method for two constraints is used when two constraint curves exists.

Assuming all functions are differentiable, let $f(x, y, z)$, constrained to both $g(x, y, z) = k$ and $h(x, y, z) = m$, have a local extrema at (a, b, c). Then $\nabla g(a, , b, c)$ and $\nabla h(a, b, c)$ are both orthogonal to the curve of their intersection and so is $\nabla f(a, b, c)$. That is, the three gradients are on the same plane, this can be expressed as $\nabla f(a, b, c) = \lambda \nabla g(a, b, c) + \mu \nabla h(a, b, c)$.

Questions: Find the point closest to the origin on the line of intersection of the planes $y + 2z = 12$ and $x + y = 6$.

(a) What is the objective function, f, and what are the constraints, g and h?

(b) Write five Lagrange multiplier equations, including the two constraints.

(c) Solve for the point. Find the distance of the point from the origin.

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GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

Upload your practice exam in your own hand writing. The points will be awarded in the individual work for the completion; semi correct work is required. No additional work is needed!