Week 8-Lab 1: Worksheet 11: Section 15.1

I said: " I want to tell you a story of my privileges. The year I started second grade, our school district started morning and afternoon shifts in our school building; elementary school students went to school for the afternoon shift and middle school students for the morning shift. This caused all sorts of problems for parents including my mom who was a middle school math teacher. Finding a baby sitting arrangement, in such a short notice, was impossible so I spent my mornings either in my mom's classroom or in the teacher lounge for most of that year. I sat in the back of class and did my homework but also listened to Algebra and Geometry. I didn't know multiplication so I retained some Algebra but a surprising amount of Geometry knowledge. I was bitter about that arrangement for many years; I was attending my mom's classes to avoid running errands for the associate principal in the teacher lounge. So little I knew those classes were the biggest influences in my life; I learned how to parse information even when I didn't have the prerequisite knowledge. I also learned not be afraid of not knowing the entire picture in classroom because it will fall in place eventually."

Iterated Integrals: Fubini's Theorem

If $f(x,y)$ is continuous on a rectangle $R = [a,b] \times [c,d]$ then

$$\iint_R f(x,y)\,dA = \int_{x=a}^{x=b}\left(\int_{y=c}^{y=d} f(x,y)\,dy\right)dx$$
$$= \int_{y=c}^{y=d}\left(\int_{x=a}^{x=b} f(x,y)\,dx\right)dy$$

The integrals on the right-hand-side are called **iterated integrals**.

To evaluate the iterated integral $\int_a^b \int_c^d f(x,y)\,dy\,dx$:

(i) Evaluate the inner integral for y by treating x as a fixed parameter. The result of $\int_c^d f(x,y)\,dy$ is a **function** of x.

(ii) Then evaluate the integral with respect to x.

Split Double Integrals

When a continuous function $f(x,y) = g(x)h(y)$ is a product of x-only and y-only on $R = [a,b] \times [c,d]$, then $\iint_R g(x)h(y)\,dA = \left(\int_b^a g(x)\,dx\right)\left(\int_c^d h(y)\,dy\right)$.

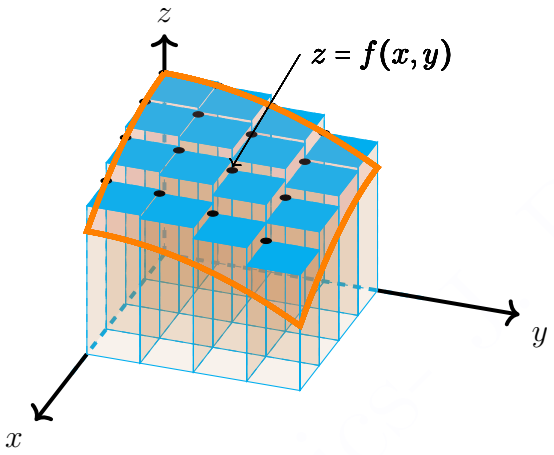
Group Work Portion of the Worksheet

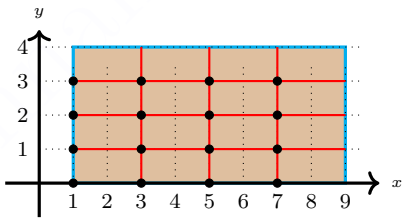
Names: _____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** You will see this problem on your Achieve assignments. You can discuss what it means when sampling points are lower left, upper left, lower right and upper right end points. This gives you a view of sample points in the domain and what it means to find the height of the rectangles at each sample point.

   Questions: Approximate $\int\int_{[1,9]\times[0,4]} f(x,y)\,dA$ as a Riemann sum $S_{4,4}$ using the lower left end points.





$f(1,4)=5$	$f(3,4)=5$	$f(5,4)=4$	$f(7,4)=3$	$f(9,4)=3$
$f(1,3)=6$	$f(3,3)=6$	$f(5,3)=5$	$f(7,3)=4$	$f(9,3)=3$
$f(1,2)=6$	$f(3,2)=6$	$f(5,2)=6$	$f(7,2)=5$	$f(9,2)=4$
$f(1,1)=7$	$f(3,1)=7$	$f(5,1)=6$	$f(7,1)=6$	$f(9,1)=5$
$f(1,0)=7$	$f(3,0)=7$	$f(5,0)=6$	$f(7,0)=6$	$f(9,0)=5$

2. **Background Story:** Discuss with your friends what are the inner and outer integral. Discuss when you can separate the integral and how to do that.

Questions:

(A) What is the inner integral in $\displaystyle\int_1^5 \int_2^8 \left(\frac{x}{y} + \frac{y}{x}\right) dy\, dx$.

(B) Evaluate the inner integral of $\displaystyle\int_1^5 \int_2^8 \left(\frac{x}{y} + \frac{y}{x}\right) dy\, dx$.

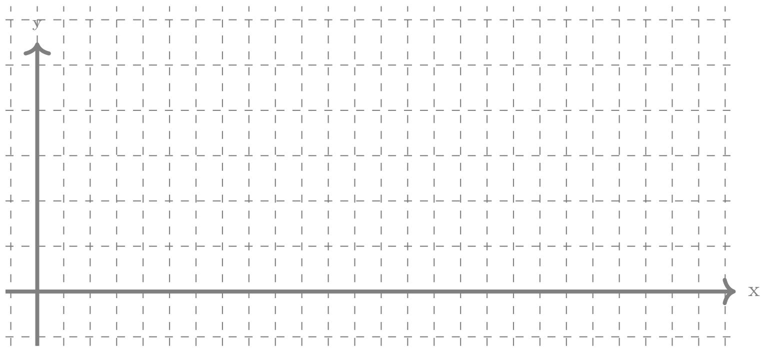
(C) Evaluate the iterated integral $\displaystyle\int_1^5 \int_2^8 \left(\frac{x}{y} + \frac{y}{x}\right) dy\, dx$.

(D) True or False: $\displaystyle\int_1^5 \int_2^8 \left(\frac{x}{y}\right) dy\, dx = \left(\int_2^8 \left(\frac{1}{y}\right) dy\right)\left(\int_1^5 x\, dx\right)$. Explain your answer.

3. **Background Story:** This question is a review of solving inequalities from precalculus and is useful for section 15.2.

Questions:

(A) Shade the region bounded by $x = 0$, $y = \sqrt{x}$, and $y = 5$. (Graph the curves and use test points.)



(B) Shade the solution to $\begin{cases} 0 \leq x \leq 25 \\ \sqrt{x} \leq y \leq 5 \end{cases}$. (Use an arrow from the lower curve to higher curve/line.)

(C) Shade the solution to $\begin{cases} 0 \leq x \leq y^2 \\ 0 \leq y \leq 5 \end{cases}$. (Use an arrow from the lower curve to higher curve/line.)

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: _____

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

Both individual questions are posted in Worksheet 12.