Week 9-Lab 1: Worksheet 13: Section 15.3

They said: "Oh! That spring break was shorter than I expected!" I said: "I agree! I am happy to see you again though!"
Also, in spirit of missing π day, I quote an unknown math enthusiast joke: They said: "Pi r squared.", Baker said: "No pies are round; cakes are square."

Triple integrals:

Use geometry of the picture to find two surfaces that bound the inner variable. This step does not require eliminating a variable between two equations.

After the inner integral is set. Find the reflection of the solid on the coordinate plane of the two variable. Treat the two outer integrals as a double integral on the region of reflection. This step may require eliminating a variable between two equations.

Non-simple Regions:

Divide into two or more simple regions and set one iterated integral for each region.

Some concrete applications of triple integrals:

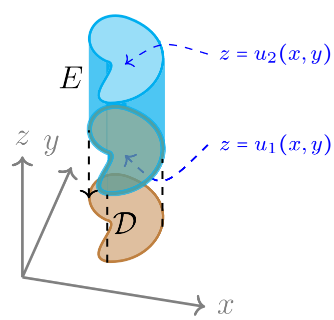
$$\text{Volume of a solid region } W = \iiint_W 1\, dV$$

$$\text{Average value of } f \text{ on } W = \frac{\iiint_W f(x,y,z)\, dV}{\text{volume}(W)}$$

$$\text{Total amount of "stuff" in } W = \iiint_W \delta(x,y,z)\, dV \text{ (Total mass is an example.)}$$

where $\delta(x,y,z)$ = density at point (x,y,z)

A solid E is called **z-simple** if the **z** coordinate is bounded by two continuous functions of x and y.



$$D = \left\{(x,y) \;\middle|\; (x,y,z) \in E\right\}$$

$$E = \left\{(x,y,z) \;\middle|\; \begin{array}{l}(x,y) \in D,\\ u_1(x,y) \le z \le u_2(x,y)\end{array}\right\}$$

$$\iiint_E f(x,y,z)\, dV = \iint_D \left(\int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\, dz\right) dA$$

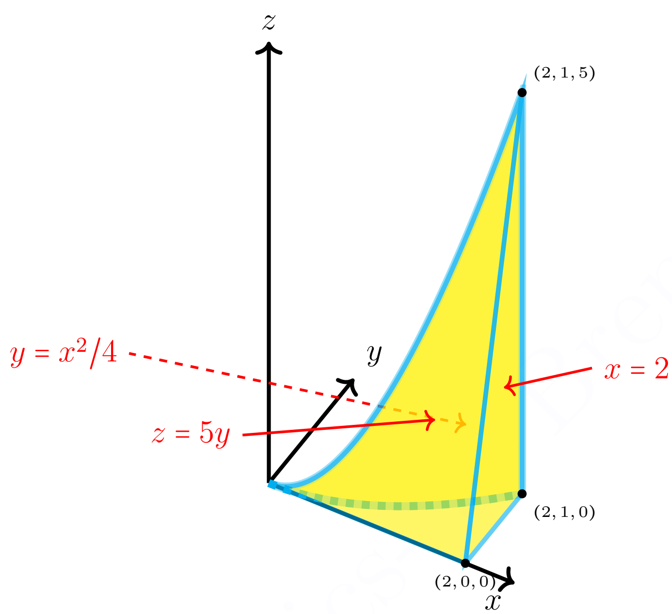
Calculating the inner integral produces a function of x and y. The middle and outer integrals are set up as a double integral.

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. (A) Consider $\int_0^2 \int_0^{x^2/4} \int_0^{5y} f(x,y,z)\,dz\,dy\,dx$. Explain to each other why the following figure is the solid of the integration. (You do not have to write this part down.)



(B) Which of the following is an iterated integrals that is equal to $\int_0^2 \int_0^{x^2/4} \int_0^{5y} f(x,y,z)\,dz\,dy\,dx$. Explain why the other three answers are **NOT** correct.

(A) $\int_0^1 \int_{2\sqrt{y}}^2 \int_0^{5x^2/4} f(x,y,z)\,dz\,dx\,dy$

(B) $\int_0^1 \int_0^{2\sqrt{y}} \int_0^{5x^2/4} f(x,y,z)\,dz\,dx\,dy$

(C) $\int_0^1 \int_{2\sqrt{y}}^2 \int_0^{5y} f(x,y,z)\,dz\,dx\,dy$

(D) $\int_0^1 \int_0^{2\sqrt{y}} \int_0^{5y} f(x,y,z)\,dz\,dx\,dy$

(C) On the above figure, mark the two surfaces bounding the most inner integral in the order $dy\,dz\,dx$.

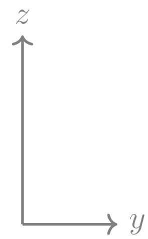
(D) Write four iterated integrals that are equal to $\int_0^2 \int_0^{x^2/4} \int_0^{5y} f(x,y,z)\,dz\,dy\,dx$ in these orders:

(i) $dx\,dz\,dy$:

The inner integral bounds are:

$\square \leq x \leq \square$

The region for the two outer integrals:



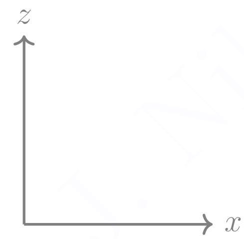
$$\int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} f(x,y,z)\,dx\,dz\,dy$$

(iii) $dy\,dz\,dx$:

The inner integral bounds are:

$\square \leq y \leq \square$

The region for the two outer integrals:



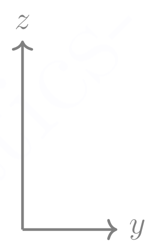
$$\int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} f(x,y,z)\,dy\,dz\,dx$$

(ii) $dx\,dy\,dz$:

The inner integral bounds are:

$\square \leq x \leq \square$

The region for the two outer integrals:



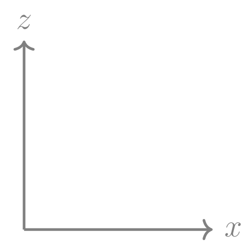
$$\int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} f(x,y,z)\,dx\,dy\,dz$$

(iv) $dy\,dx\,dz$:

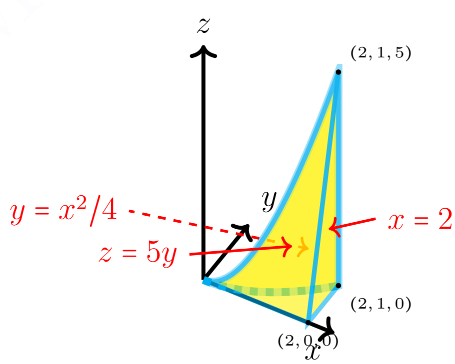
The inner integral bounds are:

$\square \leq y \leq \square$

The region for the two outer integrals:



$$\int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} f(x,y,z)\,dy\,dx\,dz$$

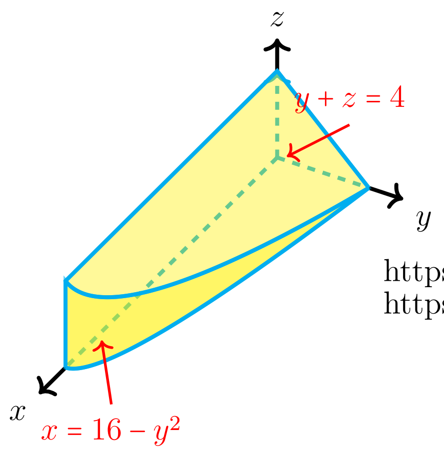


Video: https://youtu.be/4HAw9ttQvoE

Gegebra Sheet: https://www.geogebra.org/m/mnw4gbyz

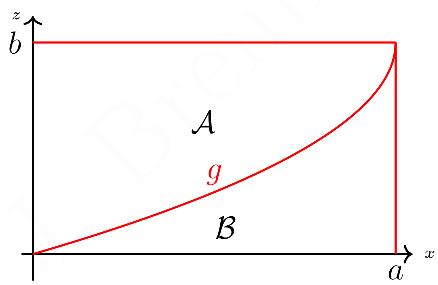
2. **Background Story:** The following solid is not simple in y-direction.

   Questions: Use **triple integrals** to find the volume of the solid in the first octant which is bounded by the coordinate planes, $y + z = 4$, and $x = 16 - y^2$.



https://www.geogebra.org/m/vcbdfumh
https://www.geogebra.org/m/czabhq5p

   (A) The solid integral is non-simple y-direction. The graph shows the two regions, $\mathcal{A}$ and $\mathcal{B}$, where the outer integrals can be set up as two y-simple solids. Find curve g. Find the values a and b. Find the bounds in y-direction.



   (B) Set up the integral in the non-simple y-direction using Part (A). (Either use $dy\,dx\,dz$ order or $dy\,dz\,dx$ order.)

   (C) Set up the integral in $dz\,dx\,dy$ order or $dz\,dy\,dx$ order.

   (D) Evaluate the volume.

Videos: https://youtu.be/-CwcJ1SrJ9E and https://youtu.be/s7AAcbL6kQU

3. **Background Story:** Sometimes, even if you find a way to express the most inner integral as a simple integral, the outer double integrals region is not simple. Note that, this problem can be solved with methods in Section 15.2.

   Questions: Find the volume of the solid bounded below $z = 3$ and above the plane containing the points $(0, 0, 4)$, $(0, 4, 0)$, and $(4, 0, 0)$ in the first octant.

   The solid: https://www.geogebra.org/m/sfnk3qfk

GroupWork Rubrics:

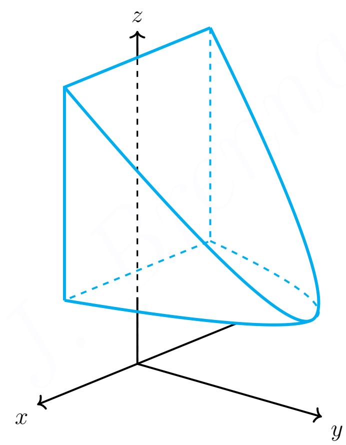
Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

4. *(3.5 points) Find the volume of the solid enclosed by* $z = 7$, $y = 0$, $y = 16 - x^2$, *and* $z + y = 23$.

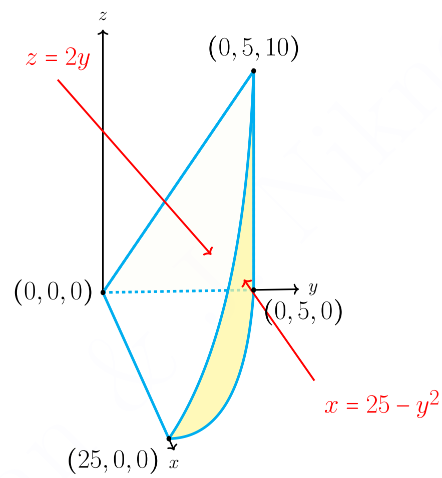


5. *Set up, do* **not** *solve, three equivalent iterated triple integrals representing*

$$\int_0^5 \int_0^{25-y^2} \int_0^{2y} f(x,y,z)\,dz\,dx\,dy$$

in orders

(A) (1 point) $\displaystyle\int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} f(x,y,z)\,dz\,dy\,dx$



(B) (1 point) $\displaystyle\int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} f(x,y,z)\,dx\,dz\,dy$

(C) (1.5 points) $\displaystyle\int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} f(x,y,z)\,dy\,dz\,dx$