# Week 11-Lab 1: Worksheet 16: Sections 15.5, 13.1

*They said: "When will I use calculus in engineering?" I said: "My first computing arc length of a helix was when I was a freshman and starting my second semester of college. In Multivariable calculus, we were covering limits and continuity. Unrelated to the school, I was put on the spot in front of three experienced civil engineers, one of whom was about to catch a flight to another city; they wanted to measure the length of a spiral so they can evaluate their contractor's cost assessments. They had measurements of height of each teeth of the spiral and the radius of cylinder containing the spiral. I used this method to compute the length: https://www.geogebra.org/m/bnwtjnfa"*

**Moments and Center of Mass:** The **mass** of a lamina $D$ with mass density $\rho$ is given by formula $m = \iint_D \rho(x,y)dA$.

The **moments** $M_x$ and $M_y$ of a lamina measure how balanced it is with respect to the $x$- and $y$-axes.

$$M_x = \iint_D y\delta(x,y)\,dA \qquad M_y = \iint_D x\delta(x,y)\,dA$$

where $D$ is the region occupied by the lamina.

The coordinates $(\overline{x},\overline{y})$ of the center of mass are

$$\overline{x} = \frac{M_y}{m} = \frac{1}{m}\iint_D x\delta(x,y)\,dA$$

$$\overline{y} = \frac{M_x}{m} = \frac{1}{m}\iint_D y\delta(x,y)\,dA$$

## Vector-Valued Functions

A **scalar function** is a function whose output is a scalar.

A **vector function** (or **vector-valued function**) is a function whose output is a vector.

## How to Recognize their Graphs

To recognize a space curve, eliminate one of the coordinates at a time; graph that projection. For example, for $\vec{\mathbf{r}}(t) = \langle x(t),\, y(t),\, z(t)\rangle$; look at the projection $\langle x(t),\, y(t)\rangle$ on $xy$-plane, look at the projection $\langle x(t),\, z(t)\rangle$ on $xz$-plane and look at the projection $\langle y(t),\, z(t)\rangle$ on $yz$-plane. This should give you some information about the curves if you are choosing among a few graphs.

If the projections are

## Group Work Portion of the Worksheet

**Names:** ______________________________________________________________

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. Find and sketch the projections of the curve on the three coordinate planes.

$$\vec{r}(t) = \langle \sin(t), t, 2\cos(t) \rangle$$

2. **Background Story:** Sketching the $xy$, $yz$ and $xz$ projections of the space curve helps you understand the curve. Try that for the following parametrizations and match them with the correct space curve.

   **Questions:** Match the curve parameterized by each vector-valued function. (Enter I, II, III, IV, V, or VI.)

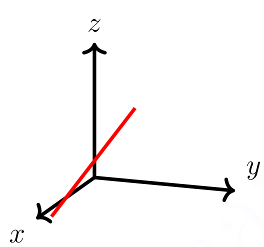
   (a) $\vec{\mathbf{r}}(t) = \langle \cos(t), \sin(t), \sin(4t) \rangle$

   (b) $\vec{\mathbf{r}}(t) = \langle t, \cos(t), \sin(t) \rangle$ and $t \geq 0$

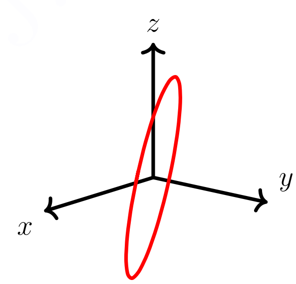
   (c) $\vec{\mathbf{r}}(t) = \langle \cos(t), \sin(t), 4\sin(t) \rangle$

   (d) $\vec{\mathbf{r}}(t) = \langle 3 + 2\cos(t), 1 + 4\cos(t), 2 + 5\cos(t) \rangle$

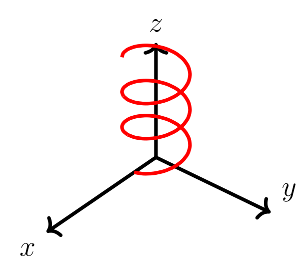
   (e) $\vec{\mathbf{r}}(t) = \langle \cos(t^3), \sin(t^3), t^3 \rangle$ and $t \geq 0$




(I)




(II)

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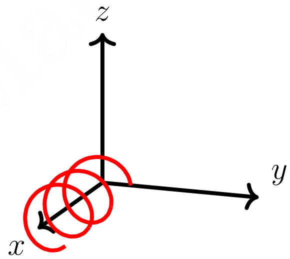
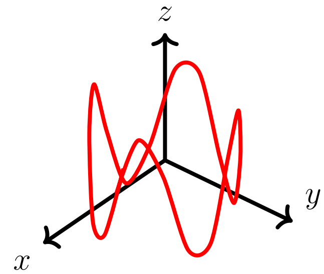
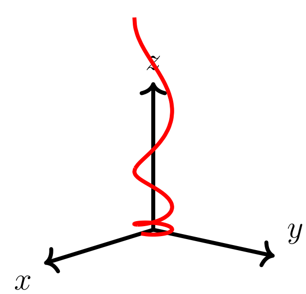



(IV)




(V)




(VI)

Video: https://youtu.be/G1AUgv2kzeA

3. **Background Story:** Helices show up in different area of science and engineering. They also are some of the few types of curves which have closed form arc length functions.

**Questions:**

One method of creating a helix is drawing equidistant parallel line segments of the same length where each segment starts at the same edge of a rectangular page and ends on the parallel edge; with top line segment and the bottom line segment are connected to two corners of the page. Then rolling the page into a cylinder matching each end of line segment with beginning of the next line segment.
https://www.geogebra.org/m/bnwtjnfa
We want to create a helix using the above method where the parameterization of the helix is

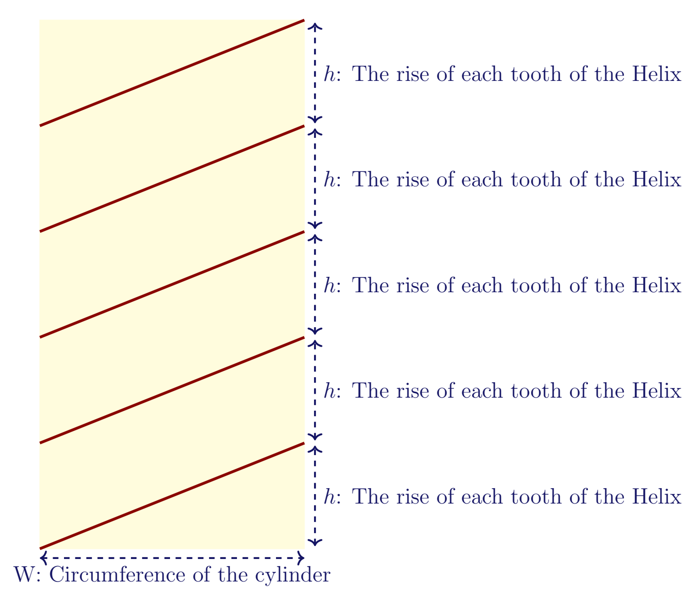
$$\vec{\mathbf{r}}(t) = \langle 5\cos(t), 5\sin(t), 3t\rangle \text{ for } 0 \leq t \leq 10\pi.$$

(A) What is the rise of each tooth of the helix (The distance between the start points of two consecutive line segments)?

(B) What is the height of the rectangular page?

(C) What is the width of the rectangular page?

(D) Use Pythagorean Theorem to compute the arc length of the Helix.




Rectangular page

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

## Individual Portion of the Worksheet

**Name:** ________________

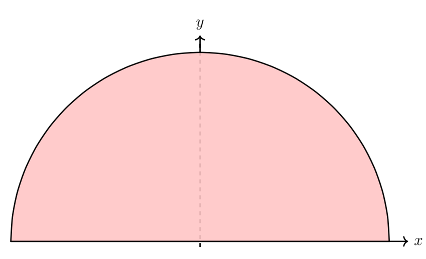
Upload this section individually on canvas or turn it in to your instructor on the $2^{\text{nd}}$ lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

4. Mass density distribution of a lamina with equation $x^2 + y^2 \leq 25$ and $y \geq 0$ (upper half of a disk) is given by $\sigma(x, y) = 16$.

   (A) *( 0.75 points) Find the mass of the lamina.*

   *(B) ( 2.5 points) Find the center of the gravity of the lamina.*

   *(C) ( 0.25 points) Does the answer in part (B) make sense?*



5. **Background Story:** Parametrization of a curve in 3-D is not that different from parametrization in 2-D. In fact, we are asking you to try a 2-D parametrization for two variables and then solve for the last component. Also, Geogebra ia a fantastic free tool that you can use. https://www.geogebra.org/m/dnac

**Questions:** Find a parametrization of the curve that represents the curve of intersection of each pair of surfaces.

(A) *(1.75 points)*

$$x^2 + y^2 = 1 \qquad z = x^2 - y^2$$

(B) *(1.75 points)*

$$\frac{x^2}{25} + \frac{z^2}{100} = 1 \qquad x + y = 6$$