Week 2-Lab 1 or 2: Worksheet 2: Vector Review

I asked them to make a new I said / they said for me and they wrote the following: They said: "I am too tired and can't even think about my Achieve. I'm going home to take a nap." I said: "If you go to the math help room for two hours, you can probably get a lot of your assignments done."

Equations of a Line in 3-Space

Let L be a line in $\mathbb{R}^3$, with direction vector $\vec{\mathbf{v}} = \langle v_1, v_2, v_3 \rangle$, containing a point $P_0 = (x_0, y_0, z_0)$.

Vector form $\quad \vec{\mathbf{r}} - \vec{\mathbf{r}}_0 = t\vec{\mathbf{v}}$ for all t

$\vec{\mathbf{r}}(t) = \langle x_0 + tv_1,\ y_0 + tv_2,\ z_0 + tv_3 \rangle$

Parametric form $\quad x = x_0 + tv_1,\ y = y_0 + tv_2,\ z = z_0 + tv_3$

Symmetric form $\quad \dfrac{x - x_0}{v_1} = \dfrac{y - y_0}{v_2} = \dfrac{z - z_0}{v_3}$

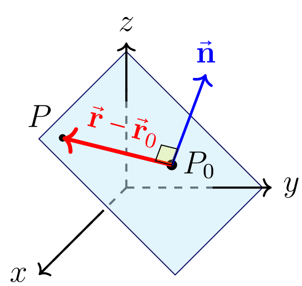
(provided $v_1, v_2, v_3 \neq 0$)

The symmetric form consists of two equations on x, y, z, with no parameter.

The vector form and parametric form are more or less the same.

Equations for Planes

$P_0(x_0, y_0, z_0)$: point in $\mathbb{R}^3$
$\vec{\mathbf{r}}_0 = \langle x_0, y_0, z_0 \rangle$
$\vec{\mathbf{n}} = \langle n_1, n_2, n_3 \rangle$: nonzero vector



Then there is a unique plane F that passes through P_0 and is orthogonal to $\vec{\mathbf{n}}$.

Let $P(x, y, z)$ be a general point on the plane F and let $\vec{\mathbf{r}} = \langle x, y, z \rangle$.

Vector equation of F $\quad \vec{\mathbf{n}} \cdot (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$

Scalar equation of F $\quad n_1(x - x_0) + n_2(y - y_0) + n_3(z - z_0) = 0$

The vector $\vec{\mathbf{n}}$ is called a **normal vector** to F.
Any nonzero multiple of $\vec{\mathbf{n}}$ is also a normal vector to F.

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** When talking about intersections of lines and planes, it is important to connect the concepts to systems of linear equations (Yes, as in Linear Algebra but also as in College Algebra). Make the connections in the following questions.

   Questions: Consider the two planes

$$-2x + 2y + 3z = 1 \qquad\qquad 2x + y + 2z = 5.$$

   (A) Solve a system of linear equations to find the line of intersection of the two planes.

   (B) What is a direction vector of the line of intersection of the planes?

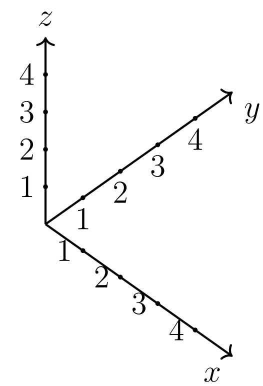
   (C) $\vec{n}_1 = \langle -2, 2, 3\rangle$ and $\vec{n}_2 = \langle 2, 1, 2\rangle$ are normal to the planes. Explain why $\vec{n}_1 \times \vec{n}_2$ is parallel to the line of intersection.

   (D) Find a vector of magnitude 4 parallel to the line of intersection of the planes using the cross product.

2. **Background Story:** Sketching in 3D is an essential component of learning vector calculus. This question is promoting that.

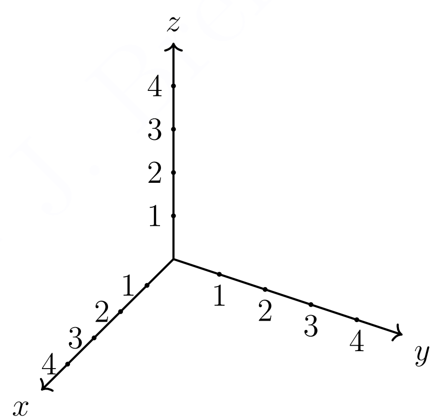
   Questions:

   (A) Sketch a triangular piece of the plane passing through points $(2,0,0)$, $(0,4,0)$, and $(0,0,6)$. Then draw a normal vector to the plane. Label all items.

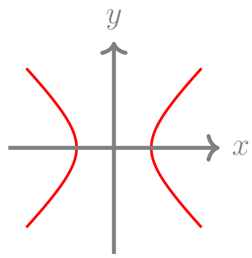


   (B) Sketch a triangular piece of plane $x + y + z = 1$ in first octant. Label x, y, z intercepts of the plane. Sketch Label all items and write your name next to them.

   (C) Sketch the plane $y = 3$. Label any intercept of the plane that exist. Sketch Label all items. Sketch two unit normal vectors to this plane.



3. Consider the hyperbola $x^2 - y^2 = 3$; which of the following can be the graph of the hyperbola? Use the intercepts of the graph to explain your answer.

(i)


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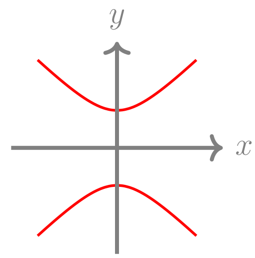

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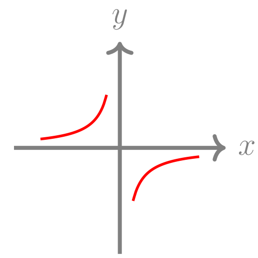

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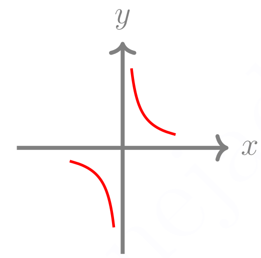
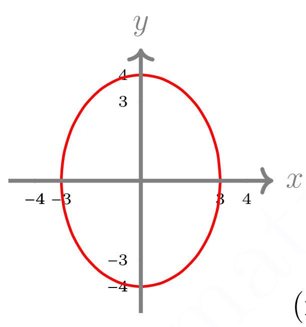


4. Consider the ellipse $9x^2 + 16y^2 = 144$; which of the following can be the graph of the ellipse? Use the intercepts of the graph to explain your answer.

(i)


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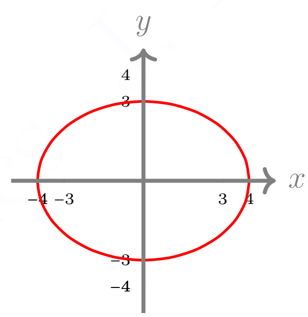

Videos:

https://mediahub.ku.edu/media/t/1_xmdimjki, https://mediahub.ku.edu/media/t/1_4rni0x07

5. **Background Story:** You may need to use trig identities multiple times to simplify.
   Questions: Simplify as much as possible.
   $(3\cos(t)\sin(s))^2 + (3\sin(t)\sin(s))^2 + (3\cos(s))^2 =$

6. **Background Story:** Here are a few PreCalculus skills we need to review.
   Questions:

   (A) Solve in terms of z: $(z-3)^2 + r^2 = 9$.

   (B) Can $3 - \sqrt{9 - r^2}$ be simplified further?

   (C) Expand the binomial: $(3x-2)^2 =$

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2^{nd} lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

7. *(1.5 points) Find an equation of the plane through point $P(2, -3, 2)$ that contains the line*

   $\vec{r}(t) = \langle 2 + 3t, t, -5 - 3t \rangle$.

8. *A symmetric equation of the line* ℓ *is* $\dfrac{x-3}{-5} = \dfrac{y+5}{2} = \dfrac{z-1}{2}$.

(A) *(0.75 points) Find a vector form (or a parametric form) of the line* ℓ*;* $\vec{\mathbf{r}}(t)$.

(B) *(0.75 point) Find the point in which the line* $\ell : \vec{r}(t)$ *intersects the plane* $6x - y + z = -66$.

9. **Background Story:** *A review of Calculus II or Precalculus.*

Questions:

(A) *(1 point) Find a parametric equation for* $\dfrac{x^2}{4} + \dfrac{y^2}{16} = 1$ *in 2D.*

(B) *(1 point) Find a parametric equation for* $(x-4)^2 + y^2 = 4$ *in 2D. What are the center and the radius of this circle?*

(C) *(1.5 points) Let* C *be a circle centered at* $(4, 0, 7)$ *of radius* 2 *in the plane* $z = 7$*; find a parametric equation for* C *in 3D.*

Video:*https://mediahub.ku.edu/media/t/1_z2jmvr2y*