Week 2-Lab 1: Worksheet 3: Section 12.6 and 14.1

They said: "What is the point of quizzes again?" I said: " To get you to study in small chunks in a low stake setting. (Each are 0.5% of your grade?!) "I also said: " Have you been doing the Diagnostic quizzes?"

Short Descriptions and Formulas

Use these notes for the group work portion of the work:

Cross-sectional curves and Level Curves

The cross-sectional curves of a surface are the intersections of the surface with planes. For **Quadric** surfaces in this course, we look at the planes $x = k$, $y = k$ or $z = k$.

Level Curves are the cross-sectional curves of the graph of two variables with plane $z = k$.

Standard form of Quadric Surfaces: $Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz + G = 0$

If a variable is missing, you have a cylinder. For example, $x^2 + y = 0$ is a **parabolic cylinder** orthogonal to xy-plane. Parallel to z-axis.

If $A, B, C > 0$, then $Ax^2 + By^2 + Cz^2 = k$ is an **ellipsoid**.

If $A, B, C > 0$, then $Ax^2 + By^2 = Cz^2 + k$; is a **cone** if $k = 0$; is a **hyperbloid** of one sheet if $k > 0$ and cross-sectional with $z = 0$ is possible; is a **hyperbloid of two sheets** when $k < 0$ and cross-sectional with $z = 0$ is not possible.

If $A, B > 0$, then $z = Ax^2 - By^2$ is a **hyperbolic paraboloid**.

If $A, B > 0$, then $z = Ax^2 + By^2$ is a **paraboloid**.

Suggested Videos:

- Use this for graphing Hyperbolas: https://youtu.be/c5dO9jKLdJ0
- Use this if you need more information on Hyperbolas: https://youtu.be/JiPxyyPcPYA
- Basic comparison of Hyperbola and ellipses: https://youtu.be/gf3x6RX68T8
- Use this for graphing Ellipses and then in Section 14.1 for inequalities: https://youtu.be/8E1exKo8TIE
- Use this for graphing ovals: https://youtu.be/dGOR2s0fUzU
- Parametrization review: https://youtu.be/mwE3m94Bd-s

Use these notes for the individual portion of the work:

Functions of Two Variables

A **function** $f : \mathbb{R}^2 \to \mathbb{R}$ **of two variables** is a rule that assigns to each ordered pair (x, y) in a set D a **unique number** denoted by $f(x, y)$.

The set D is called the **domain** of f.

The **range** of f is the set of output values of f for points in D.

$$\text{Range}(f) = \{f(x, y) \,|\, (x, y) \in D\}$$

The domain is a subset of $\mathbb{R}^2$, and the range is a subset of $\mathbb{R}$.

Often we write $z = f(x, y)$ for pairs (x, y) in D.

x and y are the **independent** variables of f. z is the **dependent** variable of f.

The graph of a function $z = f(x, y)$ is a surface in three dimensional space and it satisfies **vertical line** test.

Group Work Portion of the Worksheet

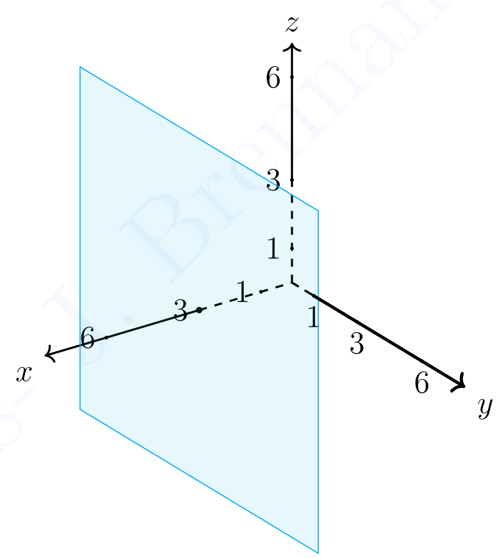
Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

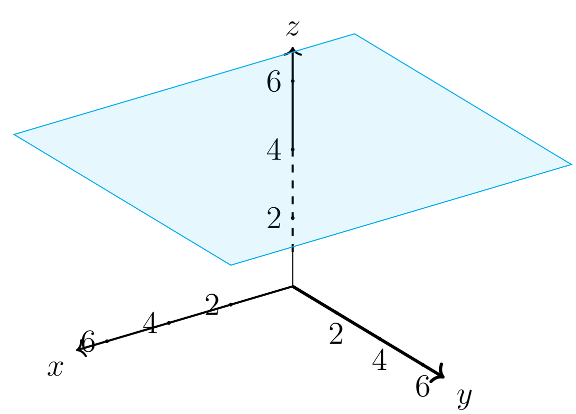
1. **Background Story:** Finding a cross-sectional of a surface means finding the intersection of the surface with a plane. These intersections, if they exist, could be point(s), lines and curves in those planes.[1] In the next question, practice graphing curves in planes in 3d.

   Questions:

   (A) Graph cross-sectional of $y^2 + z^2 = x^2$ with plane $x = 3$. That is, $y^2 + z^2 = 9$

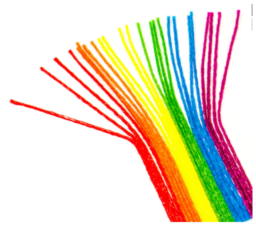


   (B) Graph cross section of $x^2 - y^2 = z$ with plane $z = 4$. That is, $x^2 - y^2 = 4$.



[1] Sometimes, they are regions in the plane.

2. **Background Story:** Graphing the Quadric surfaces is very much like computing the cross sections, forming the cross-sectional curves using aWikki Stix and then putting all Wikki Stix together.



Questions:

(A) Find the following cross sections for each of these surfaces.

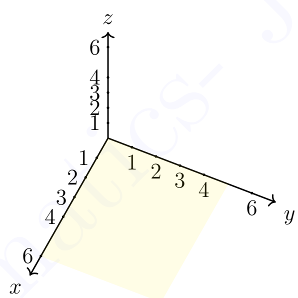
i. $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 + 0.25\,z^2 = 1$

$x = 0:$	$y = 0:$
$x = 3:$	$y = 1:$
$x = 4:$	$y = 4:$
$x = 6:$	$y = -4:$

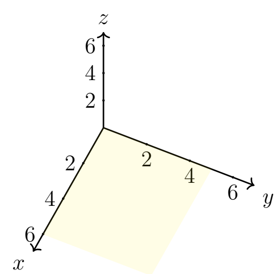
ii. $\left(\frac{x}{4}\right)^2 + \left(\frac{z}{3}\right)^2 - 0.5y = 0$

$x = 0:$	$y = 0:$
$x = 1:$	$y = 1:$
$x = -1:$	$y = -1:$
$x = 4:$	$y = 4:$
$x = 6:$	$y = -4:$

(B) Graph and label the cross sections in 3d if they exists.



(i)



(ii)

3. Identify and sketch each surface.

(A) $\left(\frac{y}{3}\right)^2 - \left(\frac{x}{4}\right)^2 - 2z^2 = 1$

Identify: ______________________

Sketch:

(B) $\left(\frac{y}{3}\right)^2 - \left(\frac{x}{4}\right)^2 - 2z^2 = 0$

Identify: ______________________

Sketch:

(C) $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 + 0.25\,z^2 = 1$

Identify: ______________________

Sketch:

(D) $\left(\frac{y}{2}\right)^2 + \left(\frac{z}{3}\right)^2 - 0.5x = 0$

Identify: ______________________

Sketch:

4. **Background Story:** Discuss multivariable functions.

Questions:

(A) In a few sentences explain what property a surface can have to make it a graph of a function $z = f(x, y)$.

(B) Give an example of a multivariable function in your field of study.

GroupWork Rubrics:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of the Worksheet

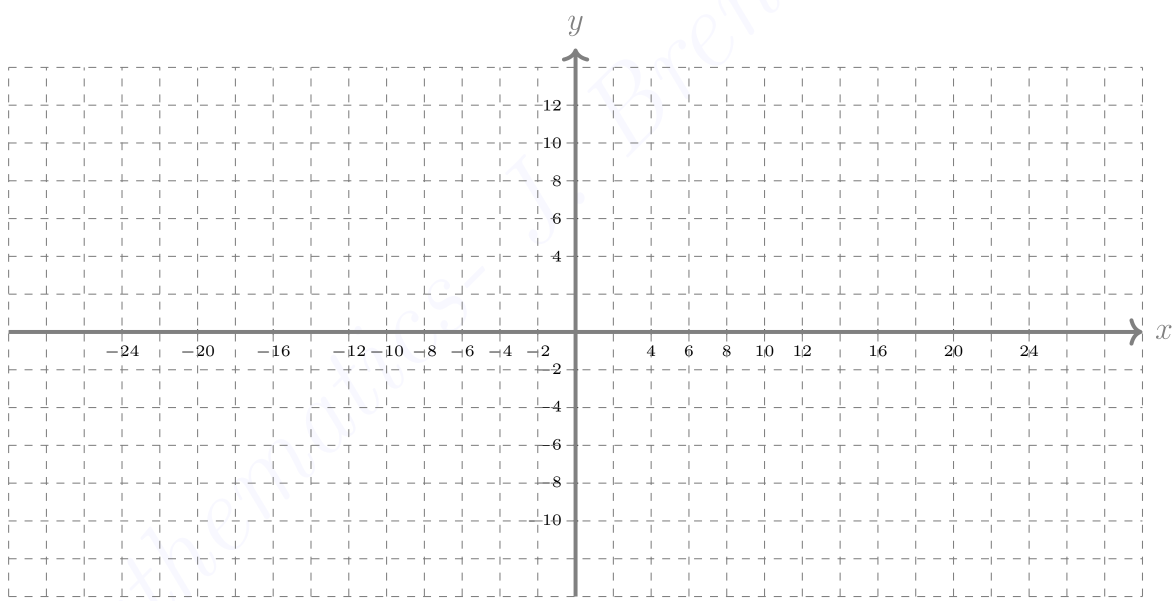
Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

5. *(1 point) Describe the level curves of the graph of the function* $f(x, y) = x^2 + 9y^2 - 144$*. Graph the level curves* $z = k$ *of the graph of* f *for* $k = -144$, 0, 432



6. *Solve each integral.*

(1) *(0.75 points)* $\int \sin(x)\cos^3(x)\,dx$

(4) *(0.5 points)* $\int_0^{2\pi} \cos^3(x)\,dx$

(2) *(0.75 points)* $\int x^3\cos(x)\,dx$

(5) *(0.5 points)* $\int \frac{x}{1+x^2}\,dx$

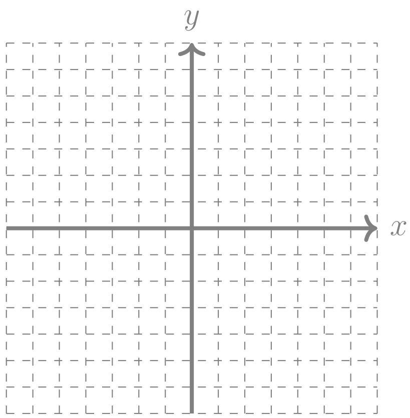
(3) *(0.5 points)* $\int \cos^2(x)\,dx$

(6) *(0.5 points)* $\int \frac{1}{9+x^2}dx$

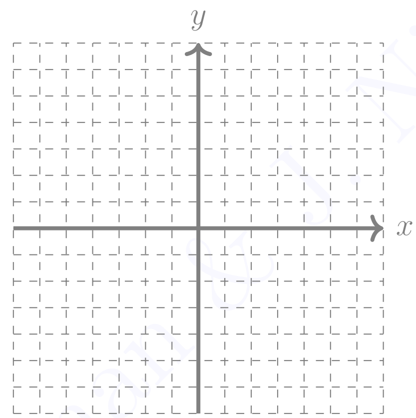
7. **Background Story:** *The domain of a function of one variable is a subset of $\mathbb{R}$. The domain of function of two variables is a subset of $\mathbb{R}^2$. To find the domain, you use the restrictions enforced by root functions, quotient functions or log functions; find a region in $\mathbb{R}^2$. Remember if you are solving inequalities switch the inequality sign to "equal to"; find the curve(s) of boundary and then divide $\mathbb{R}^2$ into two regions using the resulting curve; use test points to find the solution to inequality.*

Questions:

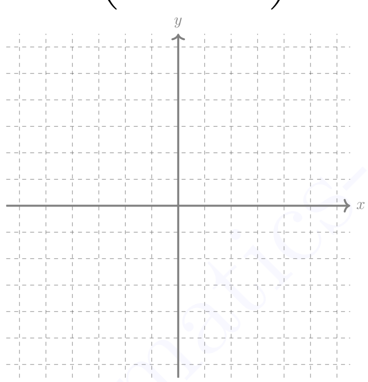
(A) *(0.5 points) Shade the domain of* $z = \sqrt{9x^2 + y^2 - 36}$.



(B) *(0.5 points) Shade the domain of* $z = \sqrt{36 - (x^2 + y^2)}$.



(C) *(0.75 points) Shade the domain of* $z = \ln\Big(6 - (x - y)\Big)$.



(D) *(0.75 points) Shade the domain of* $z = \sqrt{x - y - 6}$.