Week 3-Lab 2: Worksheet 4: Sections 11.3 and 12.7

Conversion Formulas

Conversion from polar to Cartesian coordinates:

$$x = r\cos(\theta) \qquad y = r\sin(\theta)$$

Conversion from Cartesian to polar coordinates:

$$r^2 = x^2 + y^2 \qquad \tan(\theta) = \frac{y}{x}$$

Note: Diagrams for the following are part of Problem 1 of groupwork.

Cylindrical-Cartesian Conversion

Conversion from cylindrical coordinates to Cartesian coordinates:

$$x = r\cos(\theta) \qquad y = r\sin(\theta) \qquad z = z$$

Conversion from Cartesian coordinates to cylindrical coordinates:

$$r^2 = x^2 + y^2 \qquad \tan(\theta) = \frac{y}{x} \qquad z = z$$

Spherical-Cartesian Conversion

Conversion from spherical to Cartesian:

$$x = \rho\sin(\phi)\cos(\theta) \qquad y = \rho\sin(\phi)\sin(\theta) \qquad z = \rho\cos(\phi)$$

Conversion from Cartesian to spherical:

$$\rho = \sqrt{x^2 + y^2 + z^2} \qquad \tan(\theta) = \frac{y}{x} \qquad \cos(\phi) = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

Group Work Portion of the Worksheet

Names: ____

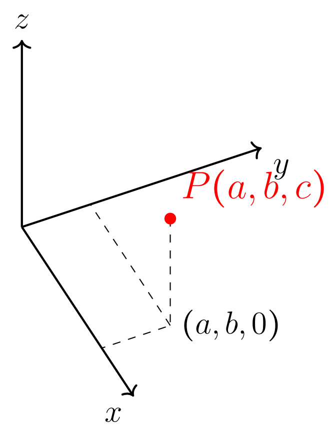
Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

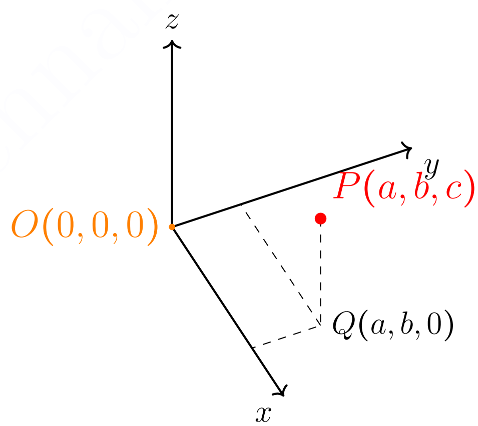
1. **Background Story:** Observe the symmetries of the cylindrical and spherical coordinates; understand the significance of using the conversions.

   Questions:

   (A) Discuss and illustrate cylindrical conversion (r, θ, z) for point P. (Draw the angle θ and line segment r.)

   (B) Discuss and illustrate spherical conversion $(\rho,\, \theta,\, \phi)$ for point P. (Draw the angles ϕ, θ, r and line segment ρ.)





   (C) Discuss finding the Spherical conversion formulas using the geometry: https://www.geogebra.org/m/wmhtgy6r. *(No writing is necessary)*

   (D) Open and share the following GeoGebra Sheets on your computer.

   (i) Discuss this **solid** in cylindrical coordinates: https://www.geogebra.org/m/pg2ctatn

   (ii) Discuss this **surface** in spherical coordinates: https://www.geogebra.org/m/vpna3c8x

   (iii) Quickly simplify! $(\rho \sin(\phi) \cos(\theta))^2 + (\rho \sin(\phi) \sin(\theta))^2 + (\rho \cos(\phi))^2$

2. Convert to a Rectangular equation. Match each Item (i)-(iv) to an Item (A)-(J). Justify your answer.

(A) A circle centered at origin with radius 3

(B) A Sphere centered at origin with radius 3

(C) A Sphere centered at $(0,0,1)$ with radius 3

(D) The xy-plane

(E) Half xy-plane

(F) Half line $x = y = 0$ and $z \leq 0$

(G) A half plane perpendicular to xy-plane

(H) A cone

(I) Upper nape of a cone

(J) Lower nape of a cone

(i) $\rho = 3$

(ii) $\phi = \pi$

(iii) $\theta = \dfrac{\pi}{3}$

(iv) $\phi = \dfrac{2\pi}{3}$

3. **Background Story:** One of the major reasons to learn spherical and cylindrical coordinates is to compute 3d integrals in Chapters 15 and 16. Those are the integrals with a high volume[1] of applications in other fields. Setting up inequalities in 3D, specially in Spherical coordinates, is a multi-step process. Practice some of those steps.

Questions:

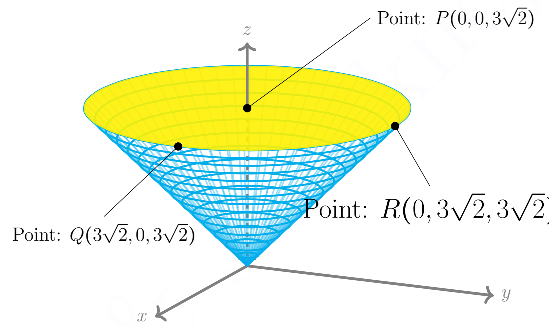
(A) Consider the solid entrapped between plane $z = 3\sqrt{2}$ and upper nape of the cone $z^2 = x^2 + y^2$.

(i) Find a point on the solid that is furthest from the origin and one that is closest.

(ii) What is the minimum and the maximum value of ρ in this piece of upper nappe of a cone? (Enter two numbers.)
$\boxed{} \leq \rho \leq \boxed{}$

(iii) Find the point in the solid that has the largest ϕ.

(iv) What is the minimum and the maximum value of ϕ for any points on this solid? (Enter two numbers.) $\boxed{} \leq \phi \leq \boxed{}$



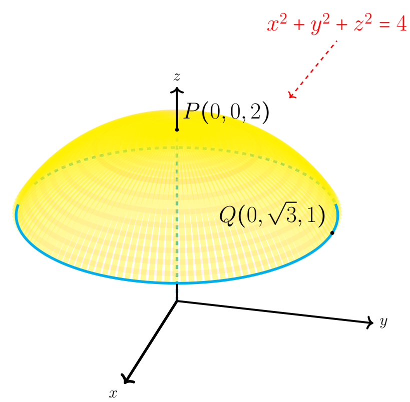
(B) Consider the solid entrapped between the sphere $x^2 + y^2 + z^2 = 4$ and $z = 1$.

(i) Find a point on the solid that is furthest from the origin.

(ii) What is the minimum and the maximum value of ρ on any point of the solid? (Enter two numbers.)
$\boxed{} \leq \rho \leq \boxed{}$

(iii) Find the point in the solid that has the largest ϕ.

(iv) What is the minimum and the maximum value of ϕ for any point on the solid? (Enter two numbers.) $\boxed{} \leq \phi \leq \boxed{}$



[1]Pun intended!

4. **Background Story:** Let's have a blast from the past! What does a surface of rotation look like? What if plane containing the curve also rotates with the curve?

Questions:

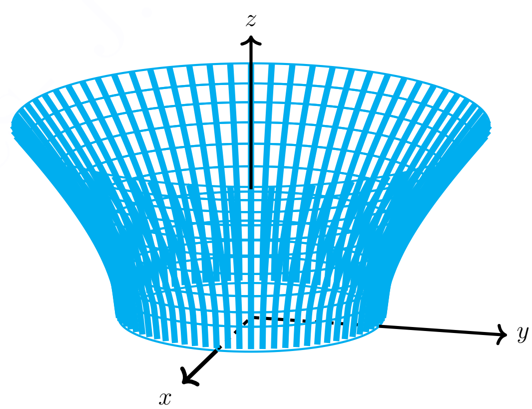
Consider the surface of $z = \sqrt{x^2 + y^2 - 1}$.

(A) Convert the Cartesian surface to Cylindrical coordinates.

(B) Algebraically find the cross section of the surface with the plane $\theta = k$; graph in rz-plane.

(C) Does answer to Part (B) change when k changes?

(D) Draw the curve of cross section of two planes $\theta = k_1$ and $\theta = k_2$ with the hyperboloid of one sheet for two values $k_1 \neq k_2$ of your choosing.



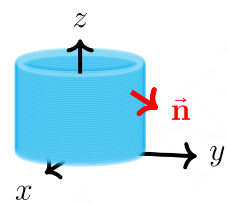
(E) Compare "this surface of rotation" is in terms of Calculus II and Calculus III notations.

5. **Background Story:** This problem is shadowing the concept of cylindrical and spherical standard unit basis. [2]

Questions:

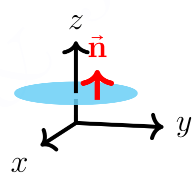
In each of the following cases $\vec{\mathbf{n}}$ is a unit vector in some direction. Compute $\vec{\mathbf{n}}$ then convert any x, y, and z appearing in $\vec{\mathbf{n}}$ to **cylindrical** r, θ, z or **spherical** ρ, ϕ, θ. For example, when $\vec{\mathbf{n}}$ is a unit vector in direction of z-axis so $\vec{\mathbf{n}} = \langle 0, 0, 1\rangle$; in this case, no x, y, z appears in $\vec{\mathbf{n}}$ and no Conversion is needed.

(A) $\vec{\mathbf{n}}$ is the **unit vector** in direction $\langle x, y, 0\rangle$, find $\vec{\mathbf{n}}$ (in terms of x, y, z); use **cylindrical** conversion formula to convert x, y, z of $\vec{\mathbf{n}}$ to r, θ, z.



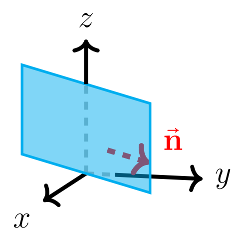
$\vec{\mathbf{n}} =$

(B) $\vec{\mathbf{n}}$ is the **unit vector** in direction of z-axis, find $\vec{\mathbf{n}}$ (in terms of x, y, z); use **cylindrical** conversion formula convert to x, y, z of $\vec{\mathbf{n}}$ to r, θ, z.



$\vec{\mathbf{n}} =$

(C) $\vec{\mathbf{n}}$ is the **unit vector** in direction orthogonal to plane containing z-axis[3], find $\vec{\mathbf{n}}$ (in terms of x, y, z); use **cylindrical** conversion formula to convert x, y, z in $\vec{\mathbf{n}}$ to r, θ, z.

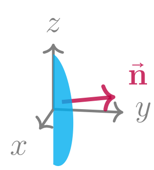


$\vec{\mathbf{n}} =$

(D) $\vec{\mathbf{n}}$ is the **unit vector** in direction $\langle x, y, z\rangle$, find $\vec{\mathbf{n}}$ in terms of x, y, z; convert x, y, z in $\vec{\mathbf{n}}$ to ρ, ϕ, θ using **spherical** conversion formula.

$\vec{\mathbf{n}} =$

(E) $\vec{\mathbf{n}}$ is the **unit vector** in direction orthogonal to plane θ =constant, find $\vec{\mathbf{n}}$ (in terms of x, y, z); use **spherical** conversion formula to convert x, y, z in $\vec{\mathbf{n}}$ to ρ, ϕ, θ.



$\vec{\mathbf{n}} =$

[2] Cylindrical Standard Unit Basis:https://www.geogebra.org/m/z32aeh6w
Spherical Standard Unit Basis: https://www.geogebra.org/m/cfybupr4

[3] That is the plane θ =constant. Note that, in electromagnetism and other area of physics, we often compute unit normal vector to a rotating rectangular regions or rotating disks. So this Part and Part (E) of this problem is helping you with that. The vectors are going to be in terms of θ only; find them however you can.

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2^{nd} lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

6. **Background Story:** A major goal of this section is to familiarize you with conversion of equations of surfaces to cylindrical or spherical coordinates. Simplify as much as possible.

   Questions:

   (A) *(2 point)* Find a spherical coordinate equation for $x^2 + y^2 + (z - 7)^2 = 49$.

   (B) *(1.5 points)* Find a spherical coordinate equation for $z = -\sqrt{\frac{1}{8}(x^2 + y^2)}$.