Week 4-Lab 1: Worksheet 5: Sections 14.2

They asked: "Why is the $\lim\limits_{(x,y)\to(a,b)}$ *notation so important?" I said: "Why is* $\dfrac{(x-1)(y+1)}{x-1} \neq y+1$*?"*

Elementary Functions: The functions we know polynomial, exponential, rational functions, trig functions and their sums, differences, products and compositions in any order. Exclude the piecewise-defined functions.

Direct Substitution: The limit of elementary function as (x,y) approach a point in the domain is done by direct substitution.

Limit of a function of two variables: In order for $\lim\limits_{(x,y)\to(a,b)} f(x,y)$ to equal L, the function $f(x,y)$ must approach L along **all** paths approaching (a,b).

If any two curves disagree, then the limit does not exist.

Warning: You **cannot** show that a limit exists by showing that two specific curves agree (because there could be a third one that disagrees).

Even showing that infinitely many curves agree might not be enough!

Case of $\frac{0}{0}$ when limit exists: Summary of what we discussed in this course:

Simplification and substitution if possible.

Directly using the squeeze theorem.

Converting to polar coordinates and then using squeeze theorem or L'Hospital rule to find a limit.

Case of $\frac{0}{0}$ when limit does not exist: Summary of what we discuss in this course:

Find two parameterized paths where limits don't agree. Examples of paths:
$(x,0): x\text{-axis},\quad (0,y): y\text{-axis},\quad (x,x): \text{line } y=x,\quad (x,x^2): \text{parabola } y=x^2$
and infinitely more.

This method is incorrect when you show that limit agrees on two paths.

What not to forget: In the entire work, don't forget to use limit notation; limit notation without an input is meaningless. Either use $(x,y)\to\cdots$ or a parameter $\to\cdots$.

Remember that if something is tending to 0 and is multiplied by something bounded, then the product tends to 0. But if something tends to zero multiplies something unbounded, you can not say that the product goes to zero(it may or may not).

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** Let's understand what goes wrong when a limit does not exist.

   Questions: consider $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{5x^2+7y^2}$.

   (A) Plug in the parametrized path $(0, y)$ in the limit. What is the value of the limit on the path?

   (B) Plug in the parametrized path $(x, 0)$ in the limit. What is the value of the limit on the path?

   (C) Are the two limits in Parts (A) and (B) equal to each other? If they are, can you say that the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{5x^2+7y^2}$ exists?

   (D) Plug in the parametrized path (x, x) in the limit. What is the value of the limit on the path?

   (E) Does the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{5x^2+7y^2}$ exist? Why or why not?

2. **Background Story:** We focus on a few methods of finding limits in this course more than others. The following problem helps you learn a new method.

   Questions:

   (A) Find an upper bound and a lower bound for $\dfrac{x^2}{3x^2+5y^2}$ when $(x,y) \neq (0,0)$. (Note one of x and y can be zero but not both at the same time.) That is, find m and M such that

   $$m \leq \frac{x^2}{3x^2+5y^2} \leq M.$$

   (B) Find two functions that squeeze $\dfrac{x^2 \sin^2 y}{3x^2+5y^2}$.[1]

   (C) Find the limit if it exists or show that it does not exist.

   $$\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2 y}{3x^2+5y^2}$$

   (D) Note that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2 y}{3x^2+5y^2}$ is the same on two paths $x=0$ and $y=0$. That is, $\displaystyle\lim_{x\to 0} \frac{0}{3x^2} = 0$ and $\displaystyle\lim_{y\to 0} \frac{0}{5y^2} = 0$. Is this enough to argue that the limit exists?

3. **Background Story:** Unrelated to the rest of the problems, in lecture, in Section 14.2, you saw $-|x| \leq x \leq |x|$.

   Questions:

   (A) Discuss among yourself why $-|x| \leq x \leq |x|$ is true but $-x \leq x \leq x$ is not.

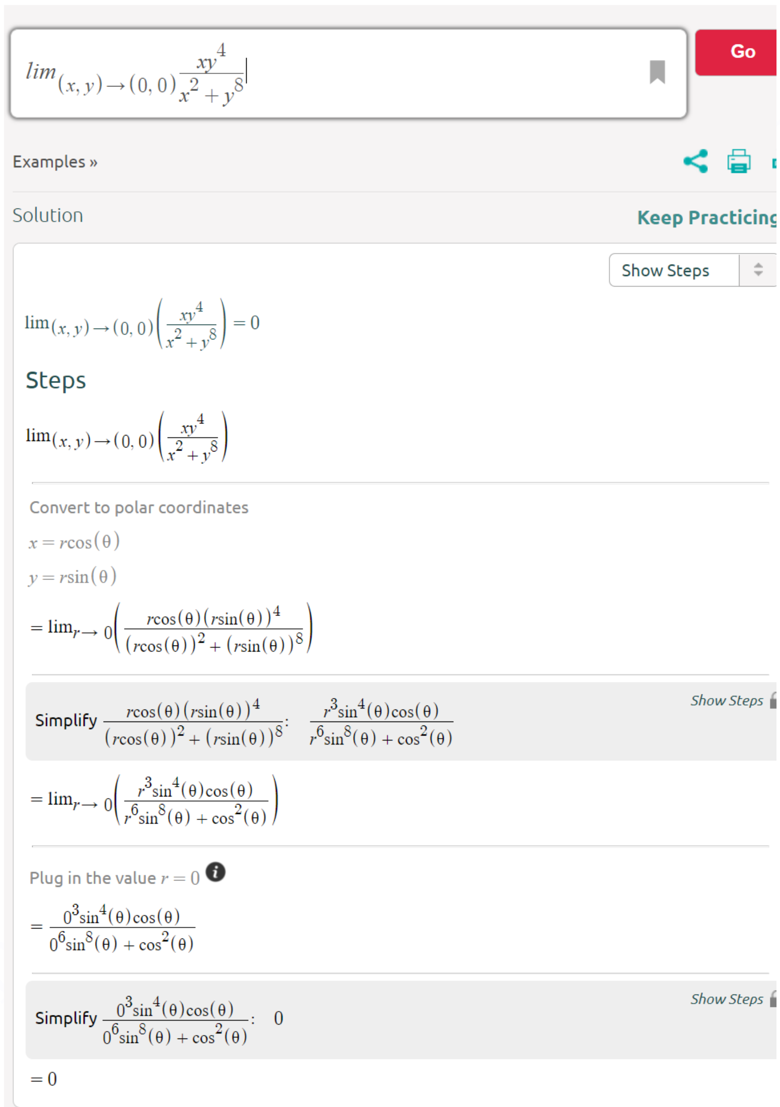
   (B) Find a lower bound and an upper bound for $6\sin(x)\cos(x) + \sin^2(x)$. That is find, m and M such that $m \leq 6\sin(x)\cos(x) + \sin^2(x) \leq M$.

[1] $f(x,y) = 0$ can be one of the functions.

4. **Background Story:** The existential question of many students is why do we need to learn math when computers can compute. The following points out an error made by one of the favorite symbolic algebra program out there. We use another program to help you visualize the error and then we ask you to find the correct value.

Questions: We use the Symbolab website to compute $\lim_{(x,y)\to(0,0)} \dfrac{xy^4}{x^2+y^8}$. Link

The following is the solution shown:



$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$

Go

Examples »

Solution

Keep Practicing

Show Steps

$\lim_{(x,y)\to(0,0)}\left(\frac{xy^4}{x^2+y^8}\right)=0$

Steps

$\lim_{(x,y)\to(0,0)}\left(\frac{xy^4}{x^2+y^8}\right)$

Convert to polar coordinates

$x=r\cos(\theta)$

$y=r\sin(\theta)$

$=\lim_{r\to 0}\left(\frac{r\cos(\theta)(r\sin(\theta))^4}{(r\cos(\theta))^2+(r\sin(\theta))^8}\right)$

Simplify $\frac{r\cos(\theta)(r\sin(\theta))^4}{(r\cos(\theta))^2+(r\sin(\theta))^8}$: $\frac{r^3\sin^4(\theta)\cos(\theta)}{r^6\sin^8(\theta)+\cos^2(\theta)}$ Show Steps

$=\lim_{r\to 0}\left(\frac{r^3\sin^4(\theta)\cos(\theta)}{r^6\sin^8(\theta)+\cos^2(\theta)}\right)$

Plug in the value $r=0$

$=\frac{0^3\sin^4(\theta)\cos(\theta)}{0^6\sin^8(\theta)+\cos^2(\theta)}$

Simplify $\frac{0^3\sin^4(\theta)\cos(\theta)}{0^6\sin^8(\theta)+\cos^2(\theta)}$: 0 Show Steps

$=0$

(A) Use the geogebra link https://www.geogebra.org/m/fcpkd7pq and explain why the limit is not zero.

(B) Compute the limit on path $x=0$.

(C) Compute the limit on path $x=y^4$.

(D) In symbolab, despite what it says, they are not computing the limit through squeeze theorem. Where is that error happening?

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

5. **Background Story:** The limit of a sum is still a sum of the limits. You may be evaluating one term of the limit that is of indeterminate form and another that is direct substitution.

   Questions:

   (A) *(0.5 point)* Use the direct substitution method to find the following elementary limit. $\lim\limits_{(x,y)\to(0,0)} 5e^{xy+1}$.

   (B) *(2.5 points)* Compute the following limit of indeterminate form $\lim\limits_{(x,y)\to(0,0)} \dfrac{2y^8 - x^8}{x^2 + y^2}$.

   (C) *(0.5 point)* Compute $\lim\limits_{(x,y)\to(0,0)} \dfrac{2y^8 - x^8}{x^2 + y^2} + 5e^{xy+1}$.

6. *(1.75 points) Find the limit if it exists or show that it does not exist.* $\lim\limits_{(x,y)\to(0,0)} \dfrac{4x^3y}{3x^4+2y^4}$

7. **Background Story:** *The following are some fundamental partial derivatives from Section 14.3.*

   Questions: *Consider* $f(x,y,z) = e^{12x+5y}\cos(13z)$. *Compute*

   (A) (0.25 points) $\dfrac{\partial f}{\partial x} =$

   (B) (0.25 points) $\dfrac{\partial f}{\partial y} =$

   (C) (0.25 points) $\dfrac{\partial^2 f}{\partial y^2} =$

   (D) (0.25 points) $\dfrac{\partial f}{\partial z} =$

   (E) (0.25 points) $\dfrac{\partial^2 f}{\partial z^2} =$

   (F) (0.5 points) Verify that $u = f(x,y,z)$ *is a solution to the equation* $u_{xx} + u_{yy} + u_{zz} = 0$.

   [2]

[2]Video: https://youtu.be/TZrFe-aJf_U