Week 4-Lab 2: Worksheet 6: Sections 14.3 and 14.4

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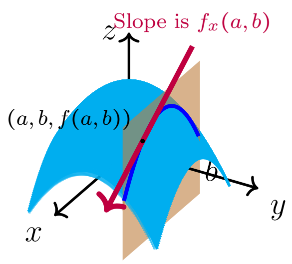
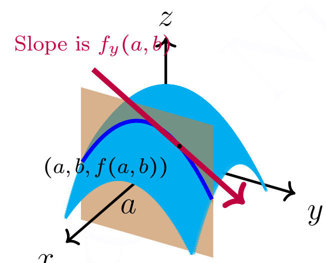
14.3: Geometry of Partial Derivatives:

The planes $x = a$ and $y = b$ intersect the surface $z = f(x,y)$ in curves $z = f(a,y)$ and $z = f(x,b)$ (respectively). The partial derivatives are the slopes of the tangent lines to the two curves.



Plane $y = b$



Plane $x = a$

https://youtu.be/b52bcTlWtFs

- The tangent line to the graph of $z = f(x,b)$ in the x-direction contains the point $(a,b,f(a,b))$ and has direction vector $\langle 1,0,f_x(a,b)\rangle$.
- The tangent line to the graph of $z = f(a,y)$ in the y-direction contains the point $(a,b,f(a,b))$ and has direction vector $\langle 0,1,f_y(a,b)\rangle$.

Higher Derivatives: $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = f_{xx}$, $\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = f_{yy}$, and $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x\,\partial y} = f_{yx}$

Clairaut's Theorem: If $f_{xy}(x,y)$ and $f_{yx}(x,y)$ are continuous, then $f_{xy} = f_{yx}$.

14.4: Differentiability:

Suppose that $(x,y) = (a,b)$ is in the domain of a function $z = f(x,y)$. We know that the tangent plane, <u>if it exists,</u> has the equation

$$L_{(a,b)}(x,y) = z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b).$$

Total Differentials: $dz = f_x(a,b)\Delta x + f_y(a,b)\Delta y$

How the Differentials and the Tangent Plane are Related:

$$\begin{aligned} L_{(a,b)}(x,y) = z &= f_x(a,b)\underbrace{(x-a)}_{\Delta x} + f_y(a,b)\underbrace{(y-b)}_{\Delta y} + f(a,b) \\ \underbrace{z - f(a,b)}_{dz} &= f_x(a,b)\underbrace{(x-a)}_{\Delta x} + f_y(a,b)\underbrace{(y-b)}_{\Delta y} \end{aligned}$$

Generalized Linearization for $\mathbf{R}^n$ (A Hyper Plane) at the point $(a_1,a_2,\cdots,a_n)$:

$$\underbrace{L_{(a_1,a_2,\cdots,a_n)}(x_1,x_2,\cdots,x_n)}_{\text{Output: } x_{n+1}} = f(a_1,a_2,\cdots,a_n) + \underbrace{f_{x_1}(a_1,a_2,\cdots,a_n)}_{\text{Partial 1}}\underbrace{(x_1-a_1)}_{\Delta x_1} + \underbrace{f_{x_2}(a_1,a_2,\cdots,a_n)}_{\text{Partial 2}}\underbrace{(x_2-a_2)}_{\Delta x_2} + \cdots + \underbrace{f_{x_n}(a_1,a_2,\cdots,a_n)}_{\text{Partial } n}\underbrace{(x_n-a_n)}_{\Delta x_n}$$

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** Partial derivatives keep showing up in many areas. Here is a business application example.

   Questions: At a distance of x feet from the beach, the price (\$) of a plot of land of area a square feet is $f(a, x)$.

   (i) What are the units of $f_a(a, x)$.

   (a) What does $f_a(1000, 470) = 7$ mean in practical terms?

   (b) What are the units of $f_x(a, x)$?

   (c) What does $f_x(1000, 470) = -4$ mean in practical terms?

   (d) Which is cheaper:

      (i) 1005 square feet that are 475 feet from the beach?

      (ii) 995 square feet that are 468 feet from the beach?

   Justify your answer.

2. **Background Story:** A common error evaluating any derivative at a point is forgetting to plug in the numbers.

   Questions: A student was asked to find the equation of the tangent plane to the surface $z = x^3 - y^2$ at the point $(x, y) = (5, 2)$. The student answered:

$$z = -2y(x-5) + 3x^2(y-2) + 121$$

   (A) At a glance, how do you know this is wrong?

   (B) What mistakes did the student make?

   (C) Answer the question correctly.

3. **Background Story:** Sometimes the linearization is a hyperplane in $\mathbb{R}^4$.

   Questions: Approximate $\sqrt{(3.14)^2+(1.93)^2+(6.07)^2}$ using a linear approximation.[1]

[1]Video: https://youtu.be/ck5GnrW1HkI

4. Find an equation of the tangent line to the graph of $f(x,y) = xe^y$ at $(5,0)$ in the y-direction.

5. Compute the first-order partial derivatives of $w(x, y, z) = \dfrac{5y}{3x + 3z}$.

GroupWork Rubrics day 2:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

All questions for individual work for this week are posted in Worksheet 5. This worksheet contains no individual question.