Week 6-Lab 1: Worksheet 8: Section 14.6

I said: "Do you know we have a help room in 651 Snow Hall where your instructors can help you? Our hours are M-F Noon-5PM. If you need outside those hours, individual or group tutoring are available here: https://learning.ku.edu/tutoring." I also said: " Don't get too much help on assignments; get as much as possible done, then ask the instructors to lead you to the next step. "

The Multivariable Chain Rule

Suppose that $z = f(x,y)$, $x = x(t)$, and $y = y(t)$ are differentiable functions. Then $z = g(t) = f(x(t), y(t))$ is a differentiable function of t and

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

In Lagrange notation, the formula is

$$g'(t) = f_x(x,y)x'(t) + f_y(x,y)y'(t)$$

Suppose $z = f(x,y)$, $x = g(s,t)$, and $y = h(s,t)$, with all functions differentiable. Then $z(s,t) = f(g(s,t), h(s,t))$ is differentiable, and

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \qquad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

For every composite function involving multiple variables, we can keep track of the Chain Rule using a tree.

Chain Rule Dependency Tree Diagram:

(1) **Begin with the variable of the top of** $\frac{d\square}{d\square}$ **or** $\frac{\partial\square}{\partial\square}$

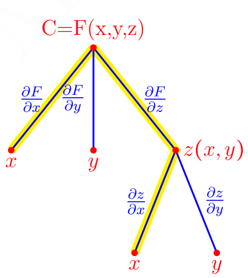
(2) **Relate the top variable to the next set of variable on the second line.**

(3) **Connect the second line variables to other variables they are immediately related to on the third line and so on until you arrive at the level that contains the bottom variable of Item (1) in this list.**

(4) **On each tree branch write** $\frac{d\square}{d\square}$ **or** $\frac{\partial\square}{\partial\square}$ **relating the two variable that branch is connecting.**

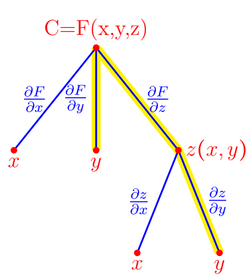
(5) **Multiply what you wrote on each branch that connects the two variable in Item (1). Then add all those numbers.**

Implicit Differentiation:



$$0 = F_x(x,y,z) + F_z(x,y,z)\frac{\partial z}{\partial x}$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}}$$



$$0 = F_y(x,y,z) + F_z(x,y,z)\frac{\partial z}{\partial y}$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{-F_y}{F_z}}$$

(Problems appear on Achieve.)

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** The chain rule tree diagrams are really good tools for memorizing all chain rule formula. [1] Practice them here. Use your Section 14.6 lecture notes.

   Questions:

   (A) Draw a tree diagram for partials g_s and g_t when $g(s,t) = f(x(s,t), y(s,t))$. (Write a variable in each node and a partial derivative or a derivative on each branch. Highlight the branches used for g_s and g_t in different color if possible.)

   (B) Discuss the next Question: Use

$$x(s,t) = 5s + 2t + 2st + 2 \text{ and } y(s,t) = 4s + 2t^2 + 3.$$

   and explain how to compute the highlighted numbers. Complete the Individual Question together if you have time.

(s,t)	x	x_s	x_t	y	y_s	y_t
$(0,0)$	2	5	2	3	4	0
$(2,3)$	30	a	b	29	4	12

[1] They take the memorization pain out of the memorization. ☺

2. **Background Story:** Here is an application of chain rule. In designing a circuit with varying elements, it is important to know how fast the current is changing. While the setting of the following problem is far from a design question, it can relay the information.

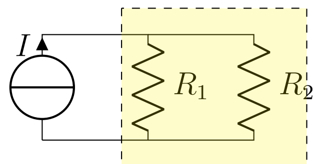
Questions: The voltage V across a circuit is given by Ohm's law

$$V = IR$$

where I is the current (in amps) flowing through the circuit and R is the resistance (in ohms). If we place two circuits, with resistance R_1 and R_2, in parallel, then their combined resistance is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Suppose the voltage drop on the resistance is 7 volts and increases at 10^{-2} volts/sec, R_1 is 3 ohms and decreases at 0.5 ohm/sec, and R_2 is 4 ohms and increases at 0.1 ohm/sec. Calculate the rate at which the current is changing.



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

GroupWork Rubrics:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

3. Suppose $f(x,y)$ is differentiable.

$$x(s,t) = 5s + 2t + 2st + 2 \quad \text{and} \quad y(s,t) = 4s + 2t^2 + 3.$$

We are given the value of the function $f(x,y)$ and it's partials at $(0,0)$ and $(2,3)$ in the table below. Let $g(s,t) = f(x(s,t), y(s,t))$.

	f	f_x	f_y
$(0,0)$	2	3	2
$(2,3)$	-4	5	4

(i) *(1 point) Compute the missing values (a and b) in the table below using the $x(s,t)$ and $y(s,t)$.*

(s,t)	x	x_s	x_t	y	y_s	y_t
$(0,0)$	2	5	2	3	4	0
$(2,3)$	30	a	b	29	4	12

(ii) (1.25 points) Find $g_s(0,0)$.

(iii) (1.25 points) Find $g_t(0,0)$.

Videos: https://youtu.be/aBKBd9VG1jA
https://youtu.be/Tn-RS0tsIfM