Differential Equations **Homework 1** **Name:**

MWF 10-10:50 or MWF 11-11:50

Show your work! Answers without supporting work will not be given credit. Print this assignment and write your work in the spaces provided.

1. Use integration by u-substitute to calculate the following antiderivatives. [1]

   (a) $\int te^{t^2}\,dt$

   $u =$ ☐

   $\int te^{t^2}\,dt =$ ☐

   (b) $\int \frac{1}{(2u+1)^2}\,du$

   $v =$ ☐

   $\int \frac{1}{(2u+1)^2}\,du =$ ☐

2. Use partial fractions to calculate the following integral.[2]

   $\int \frac{1}{y(y-4)}\,dy.$

   The partial fraction form of $\frac{1}{y(y-4)} =$ ☐

   $\int \frac{1}{y(y-4)}\,dy =$ ☐

[1] Make sure to calculate the differential of the new variable in terms of differential of the old variable. Then rewrite the integral with the new variable only. Last step is integrating with respect to the new variable.

[2] You may need to use u-substitution as well.

3. Use **integration by parts** to calculate the following antiderivatives.[3]

(a) $\int xe^{-x}\ dx$

The integration by parts: u = [ ] v = [ ]

$\int xe^{-x}\ dx$ = [ ]

(b) $\int x^2 \sin(x)\ dx$

First integration by parts: u = [ ] v = [ ]

Second integration by parts: u = [ ] v = [ ]

$\int x^2 \sin(x)\ dx$ = [ ]

4. In class we discussed that $\int e^{-x^2} dx$ is not an elementary function [4] and can only be solved numerically for each value of x[5] Give another example of such function. (Hint: you may use internet.)

[3]For one of the two, you need to use integration by parts twice.
[4]Elementary functions are functions whose closed form we know from calculus.
[5]That is any antiderivative can be given only numerically.

5. Verify that $y_1(t) = e^{-3t}$ and $y_2(t) = e^t$ are two solutions to the ode[6] $y'' + 2y' - 3y = 0$.

6. Which of the following odes are **linear** odes. Mark all that apply.

(a) $t^2 \frac{d^2y}{dt^2} + t\frac{dy}{dt} + 2y = \sin(t)$ ☐

(b) $(y^2 + y)y'' + ty' + y = 0$ ☐

(c) $\frac{d^2y}{dt^2} + \sin(y) = 0$ ☐

(d) $y''' + y'' sin(t) + y = 0$ ☐

[6]Ordinary differential equation. Note that to verify a solution, find the appropriate derivatives of each solution and then plug in the equation.

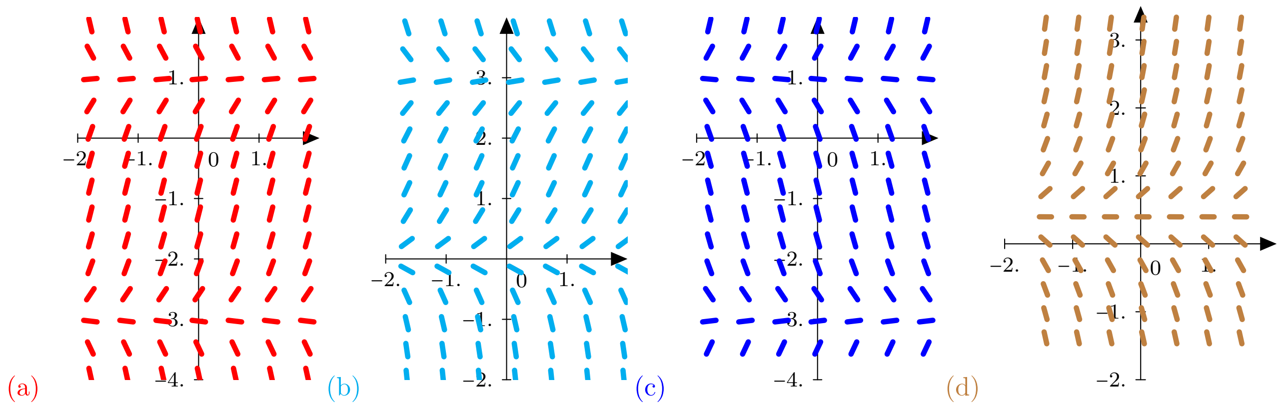
7. Match the correct the slope fields to each autonomous[7] ode. Explain why. [8]

(a) $y' = 2y - 1$

(b) $y' = (1 - y)(3 + y)$

(c) $y' = y(3 - y)$

(d) $y' = (y - 1)(y + 3)$



8. Based on the direction field determine the behaviour of **the solution** of $y' = (1 - y)(3 + y)$ as $t \to \infty$ for each of the following initial value.

(a) $y(0) = -1$

(b) $y(0) = 0.5$

(c) $y(0) = 2$

(d) $y(0) = -4$

[7]Autonomous ode is an ode where only one variable appears explicitly.

[8]You can use this app: https://www.geogebra.org/m/bgczfpUR to check your work but you have to explain your work using the equilibria and the direction around the equilibrium to.