Differential Equations **Homework 2** **Name:**
MWF 10-10:50 or MWF 11-11:50

Show your work! Answers without supporting work will not be given credit. Print this assignment and write your work in the spaces provided. This assignment covers "Linear first order equations" and some Application problems.

1. A pond initially contains $100,000$ gal of water. The water in the pond contains an undesirable chemical with concentration 0.01 ounces/gal. Water containing 0.02 ounces/gal is flowing in the pond at the rate 700 gal/hr. The mixture flows out at the same rate. Assume that the chemical is uniformly distributed throughout the pond. Let $Q(t)$ denote the amount of the chemical in the pond at time t.

   (a) Write an ode for the amount of the chemical at the pond at any time t.

   (b) What is the general solution to (a)?

   (c) What is the initial value for the **amount** of chemical in the pond.[1]

   (d) Give an expression for amount of chemical in the pond using the initial value in (c).

   (e) What is the amount of chemical in the pond in the long run?

   (f) Does the amount in part (e) depend on the initial amount of the chemical?

   (g) What is the **concentration** of chemical after one week?

   (h) After how many days the concentration in the pond is more than 0.015 ounces/gal?

[1] Concentration is the amount divided by the volume.

2. A tank initially contains Q_0 grams of dyes. The volume of the tank is V. Water that contains k gram/lit flows in at rate r lit/min. well mixed water flows out at the same rate.[2]

   (a) Find the Equation of the amount of the dye in Tank at time t.

   $Q(t) =$ $\boxed{}$

   (b) Find a formula for the limiting amount of dye using part (a).

   limiting amount= $\boxed{}$

3. Find the solution of the given **initial value** problem.
   $y' + 2y = te^{-2t}$, $y(1) = 0$.

   Standard Form: $\boxed{}$

   $\mu(t) =$ $\boxed{}$

   General Solution: $\boxed{}$

   IVP Solution: $\boxed{}$

[2] Use parameters instead of numbers. Remember that the change in quantity is equal to the incoming rate of material minus the outgoing rate of material. Q and t are the only variables.

4. Solve the following initial value problem: $y' - y = 2te^{2t}, \quad y(0) = 1.$

Standard Form: $\boxed{}$

$\mu(t) =$ $\boxed{}$

General Solution: $\boxed{}$

IVP Solution: $\boxed{}$

5. Solve the following initial value problem $ty' + 2y = \sin(t), \qquad y(\pi/2) = 1, \qquad t > 0.$

Standard Form: $\boxed{}$

$\mu(t) =$ $\boxed{}$

General Solution: $\boxed{}$

IVP Solution: $\boxed{}$

6. Solve the following initial value problem $ty' + (2-t)y = e^{3t}, \qquad y(1) = 2, \qquad t > 0.$

Standard Form: $\boxed{}$

$\mu(t) =$ $\boxed{}$

General Solution: $\boxed{}$

IVP Solution: $\boxed{}$

7. At a reception, Dad, Albert and Ava play catch on the observation deck of the Town Pavilion tower in Kansas City. To Dad's horror, their baseball rolls over the edge, squarely beaning a 180 cm tall lawyer on a lunchtime stroll along the Walnut St., $180m$ below. What happened to the lawyer? What happened to Dad? (This is a to be continued story. You will be required to solve the equation next time.)

   (a) Disregarding air resistance and assuming the initial velocity of the baseball to be zero, what is the velocity of the baseball when it hits the top of the head of the lawyer? Assume the upward direction is positive and $g = 9.8m/s^2$.

   $\boxed{}$

   (b) Now consider the problem when air drag is included in the model. Consider a model for larger and/or fast moving objects where the drag is proportional to the square of the velocity. Assume falling baseball is modeled with the following differential equation:

$$v' = -9.8 + .004v^2.$$

   Graph the direction field or phase diagram [3] for this autonomous differential equation. What is the terminal velocity in this model?

   $\boxed{}$

[3] For phase diagram, refer to section 1.6 of the text. This is a reading assignment.