Differential Equations **Homework 7** **Name:**
MWF 10-10:50 or MWF 11-11:50

Show your work! Answers without supporting work will not be given credit. Print this assignment and write your work in the spaces provided.

1. Solve $y'' + y = 9\cos(2t) + 3t\sin(2t)$.[1]

[1] Since and Cosine are present, the method of undetermined coefficient is recommended. This one is one of the rare cases that separating the forcing is not recommended. Instead, make a guess for the entire forcing function and eliminate the repeating terms.

2. Consider the equation $y'' + by' + cy = 0$.

   (a) If $b > 0$ and $c > 0$, what is the limit of the solution as $t \to \infty$?[2]

Case 1:

Case 2:

Case 3:

   (b) Describe the behaviour of the solution as $t \to \infty$ when $b = 0$ and $c > 0$.[3]

[2]Hints: Solve the characteristic equation in terms of b and c. Solve for general solution in three cases: Case 1: $b^2 - 4c > 0$, Case 2: $b^2 - 4c < 0$ and Case 3: $b^2 - 4c = 0$. You should be able to find the limit in each case as a numerical value. Also comment if oscillation, asymptotic decay or unbounded growth are present.

[3]That is, when the equation is $y'' + cy = 0$. Solve the characteristic equation in term of c. Solve for general solution.

3. Find the initial value solution to $y'' + 4y = 3\sin(2t)$ $y(0) = 2$ and $y'(0) = -1$.

4. Use the method of variation of parameter to find the general solution to $y'' - y' - 2y = 2e^{-t}$.

5. Find the general solution to $y'' + 4y' + 4y = t^{-2}e^{-2t}$. [4]

6. Find the general solution for $y'' + 4y = 12\csc(2t) \quad 0 < t < \frac{\pi}{2}$. [5]

[4] Only one of the two methods work for this one.
[5] Only one of the two methods work for this one as well.

7. Find the general solution to $y'' + y = \tan(t)$. $\quad -\frac{\pi}{2} < t < \frac{\pi}{2}$