Differential Equations **Homework 9** **Name:**
MWF 10-10:50 or MWF 11-11:50

Show your work! Answers without supporting work will not be given credit. Print this assignment and write your work in the spaces provided.

1. Prove that $\mathcal{L}(t) = \dfrac{1}{s^2}$ for $s > 0$.

2. Prove that $\mathcal{L}(te^{at}) = \dfrac{1}{(s-a)^2}$ for $s > a$

3. (a) Use Euler's identity[1] to prove $\sin(bt) = \frac{(e^{ibt} - e^{-ibt})}{2i}$.

(b) Use the formula in (a) to prove that $\mathcal{L}(\sin(bt)) = \frac{b}{s^2 + b^2}$ for $s > 0$. [2]

[1] Euler's identity is $e^{i\omega} = \cos(\omega) + i\sin(\omega)$. Do not over-think this. Start with the right hand side and use the identity to simplify to left hand side.

[2] Integrate with exponential functions only using part (a). Then use Euler identity to convert to real functions again and find the limit as $t \to \infty$.

4. Use the fact that $\mathcal{L}(\sin(bt)) = \dfrac{b}{s^2 + b^2}$ to find $\mathcal{L}(e^{at}\sin(bt))$ for $s > a$. [3]

5. Use Leibniz's rule to find $\mathcal{L}(t\sin(bt))$ for $s > 0$.

[3] Hint: Write $\mathcal{L}(\sin(bt))$ and $\mathcal{L}(e^{at}\sin(bt))$ as two integrals and compare them. Simplify the integral $\mathcal{L}(e^{at}\sin(bt))$ to look similar to $\mathcal{L}(\sin(bt))$.