Autonomous Equations:

- $y' = f(y)$ is an autonomous equation. That is, an equation where derivative with respect to t of y, $\frac{dy}{dt}$, only depends on the value of y.

- We learned earlier that this type of equations give integral curves that are parallel for each initial value on any horizontal line. *

- Let f be such that $y = a$ is a zero of a, then $y = a$ is an equilibrium solution to $y' = f(y)$.

- Now if $y' = k(y - a_1)(y - a_2)...(y - a_n)$, then we can illustrate the direction of the slops by finding the sign of f on the y-axis. We place upward or downward vectors, depending on the sign of f, above or below each equilibrium . We call this illustration, the phase diagram.

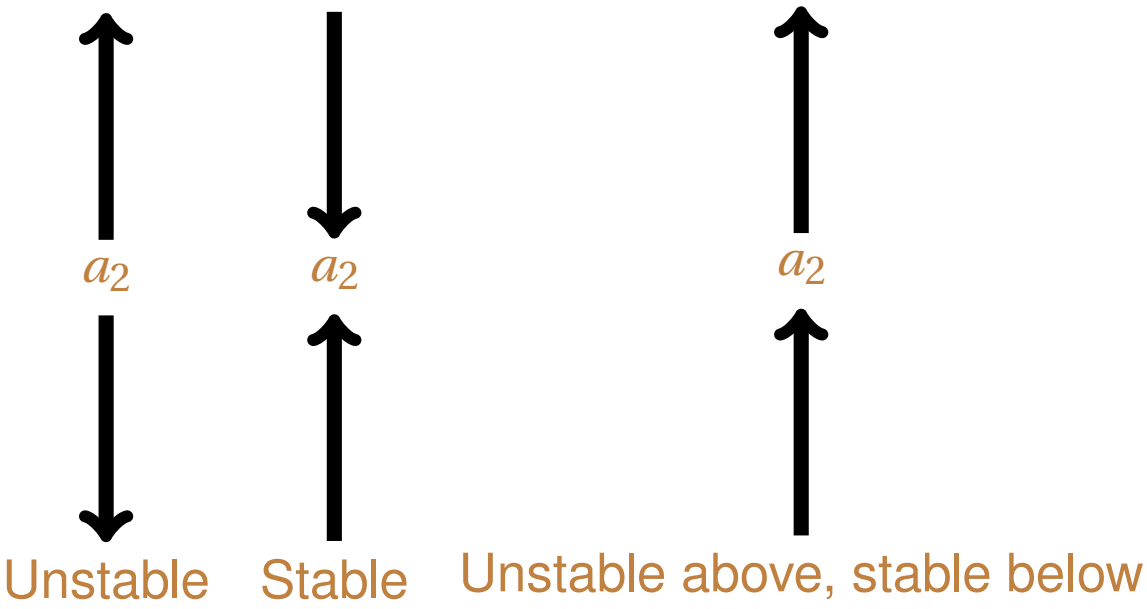
*To observe this again, use the directional field app at: https://www.geogebra.org/m/bgczfpUR

Example:

Let $f(y) = (y-1)(y-2)$ Then $f > 0$ for $(-\infty, 1)$, the lower part of diagram, $f < 0$ for $(1,2)$, the middle part of the diagram, and $f > 0$ for $(2,\infty)$, the upper part of the diagram.

Stability of Equilibria

- An stable equilibrium is where other solutions are converging to as $t \to \infty$.
- If a solution is stable, then the solution does not change drastically with small change of initial value.
- An unstable solution is the one where the solution diverges away as $t \to \infty$.
- An equilibrium can be stable from above or below and unstable from the other way.


a_2
a_2
a_2
Unstable
Stable
Unstable above, stable below