Recognition of first order methods

Here is the order I recommend that you check:

Linear : Linear equations have explicit solution so if the equation satisfies the conditions for linear, then use linear method.

Separable : Separable method is an easy way of solving first order equations.

Exact : In reality, all methods are doing is to find a way to make equations look like exact equations. But what we refer to as Exact equation is the one that can be easily be written in the form

$M(x, y)\,dx + N(x, y)\,dy = 0$, where $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$.

Bernoulli or Homogeneous: These are more computationally involved methods and take longer to do. So save these methods for last.

Appearance of Exact Equations in Other Methods:

Linear : Looking at the process, we find an integrating factor, $\mu(x) = e^{\int p(x)dx}$, to make $y' \underbrace{\mu(x)}_{N(x,y)} + \underbrace{p(x)\mu(x)y - g(x)\mu(x)}_{M(x,y)} = 0$ exact. ⚠ *

Let $M(x,y) = P(x)\mu(x)y - g(x)\mu(x)$ and $N = \mu(x)$.

Then $\dfrac{\partial M}{\partial y} = P(x)\mu(x)$ and $\dfrac{\partial N}{\partial x} = P(x)\mu(x)$.

Separable : The form is $f(y)dy = g(x)dx$.

Rewrite $\underbrace{g(x)}_{M(x,y)} dx \underbrace{-f(y)}_{N(x,y)} dy = 0$ ⚠, $M(x,y) = g(x)$ and $N(x,y) = -f(y)$.

Notice $M_{=}N_x = 0$

*Don't use this form in solving the linear equations. This is just used for the explanation seen above and **never again** will be used. This also, refers to the other danger sign.

Bernoulli and Homogeneous:

These two are substitutions that end up making the equations linear or separable and hence Exact. So really we all methods we use ends up leading us to an exact forms but we need to use them to be able to solve equations.