General Comments about Falling Objects:

In most models, drag for a larger object is considered to be proportionate to the square of velocity.* However at this point of semester, we won't be able to solve the ode of those models. That is why we use the model where drag is proportionate to the velocity.

*We have to pick out Taylor polynomial to approximate. Order two is popular but here we use order 1.

Example: Consider a falling object of mass $m = 20kg$ with drag force that is proportionate to velocity and drag coefficient $\gamma = 2kg/s$. That is, $F_d = 2|v|$. What is the equation of the motion?

Find the velocity of the object when it lands if the object was $150m$ above the ground.

Solution:

The positive direction is up.

The velocity of the the falling object will be negative with this setting.

Now air-resistance $F_d \propto |v|$

That is, $F_d = \gamma|v|$ where the drag coefficient is γ.

The direction of mg is downward and the direction of F_d is upward so:

$ma = -mg + \gamma|v|$

Since v is always negative $|v| = -v$

$mv' = -mg - \gamma v$

$(20)v' = -(20)(9.8) - 2v$

Divide both side by 20: $v' = -(9.8) - .1v$

Now The equilibrium solution is obtained by setting $v' = 0$: $-9.8 - 0.1v = 0$ which is $\boxed{v = -98}$ m/s

Exclude the equilibrium solution and solve for all other solutions:

$$\frac{v'}{-9.8 - .1v} = 1$$

$$\int \frac{v'}{-9.8 - .1v}\, dt = \int 1\ dt$$

Use the fact: $dv = v'dt$

$$\int \frac{dv}{-9.8 - .1v} = \int 1\ dt$$

Choose $u = -9.8 - .1v$ so $du = -.1dv$

LHS=$-10 \int \frac{du}{u} = -10 \ln |u| + C =$

$$-10 \ln |-9.8 - .1v| = t + C$$

so $10 \ln |-9.8 - .1v| = -t + C$

$$e^{10 \ln|-9.8-.1v|} = e^{-t+C}$$

$$\left(e^{\ln|-9.8-.1v|}\right)^{10} = e^{-t+C}$$

Since e^C is a constant:

$$|-9.8-.1v|^{10} = Ce^{-t}$$

Take the 10th root of both side: (An even root so don't forget $\pm$.)

$$|-9.8-.1v| = \pm\left(Ce^{-t}\right)^{1/10}$$

$$-9.8-.1v = \underbrace{\pm C^{1/10}}_{\text{This is complicated name for a constant}} \left(e^{-t}\right)^{1/10}$$

Replace the complicated name by C again:

$$-9.8-.1v = \pm C\left(e^{-t}\right)^{1/10}$$

Remember C can be both positive or negative:

$$-9.8 - .1v = C\left(e^{-t(1/10)}\right)$$

$$-9.8 - .1v = C\left(e^{-t/10}\right)$$

$$-9.8 - .1v = C\left(e^{-t/10}\right)$$

$$-.1v = 9.8 + C\left(e^{-t/10}\right))$$

So

$$.1v = -9.8 - C\left(e^{-t/10}\right)$$

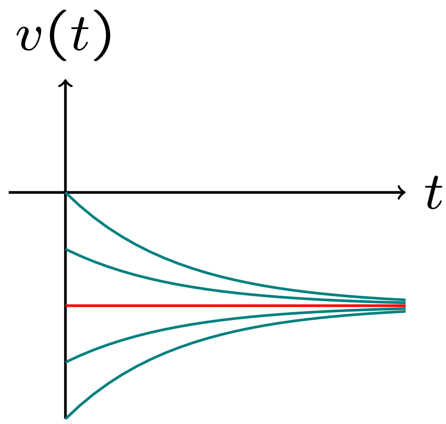
$$\boxed{v = -98 - C\left(e^{-t/10}\right)}$$ General solution

$v(0) = 0$ so $C = -98$ so $\boxed{v = -98 + 98\left(e^{-t/10}\right)}$

Initial Value Solution (IVT)

Additional Notes:

A few integral Curves For Different C's



Note that for negative value of C, the object is initially falling faster than 98 m/s but for positive values of C, the object is initially falling slower than 98 m/s. In either case as time passes, the velocity approaches -98 m/s.

And finally,

$$\boxed{v(t) = -98 + 98\left(e^{-t/10}\right)}$$ IVP solution

Now notice that $\lim_{t\to\infty} v(t) = -98$. That is, the velocity of the falling object is not going to exceed -98 m/s (Terminal velocity of the object.)

Solving for Impact Velocity:

We need to find the

Now find $x(t) = x(0) + \int_0^t v(s)ds$ or by $x(t) = \int v(t)dt$ and then using the initial values.

Note that $x(0) = 150$

Then $x(t) = 150 - 98t - 980e^{-t/10} + 980$

$$\boxed{x(t) = 1130 - 98t - 980e^{-t/10}}$$

Graph x on you calculator, set the windows correctly and find the zero of the function.

$t \simeq 6.09365448$

$v(6.094) = -98 + 98e^{-0.1\times 6.094} \simeq -44.72$