More Applications

- **Electrical Circuits**

  Consider a typical series **RC** circuit with voltage source $V(t)$. Let $I(t)$ denotes the current at time t and $Q(t)$ denotes the charge.

  Here R is the resistance in ohm (Ω) of the resistor and C is the capacitance in Farad of the capacitor.

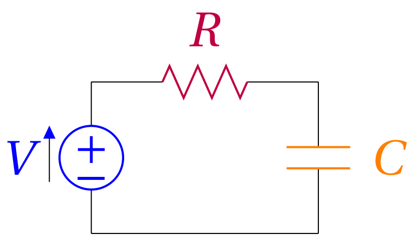
  R, C, $V(t)$ and the initial current $I(0)$ are given. The goal is to solve for $I(t)$, the current.

  Remember from Physics:

  > Voltage drop across the resistor is $R * I(t)$.
  >
  > Voltage drop across the capacitor is $\frac{1}{C} * Q(t)$.

  Voltage drop equation is: $R * I(t) + \frac{1}{C} * Q(t) = V(t)$

  Differentiate both sides to get an ode for $I(t)$:



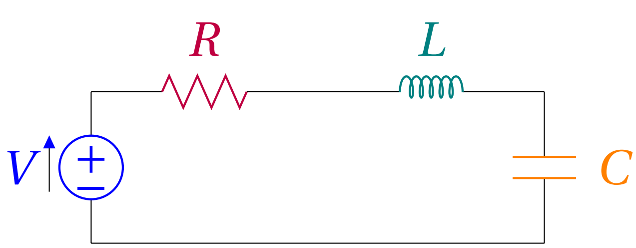
$$R * I'(t) + \frac{1}{C} * I(t) = V'(t)$$

Note that R and C are constants.

This is another example of a **linear ode** and its standard form: $\boxed{I'(t) + \frac{1}{RC} I(t) = \frac{V'(t)}{R}}$

You can add an inductance L in Henry serial in that circuit to get :

$$\boxed{L * I''(t) + R * I'(t) + \frac{1}{C} * I(t) = V'(t)}$$



- **Population Models**

  In the simplest population growth model, the rate of change of population is proportional to the population.

$\boxed{P'(t) = r(t)P(t) - b}$, $P(0) = P_0$

Where $P(t)$ is a whole number representing the number of people at time t. $r(t)$ is the growth rate, b is the rate of predation and P_0 is the initial number of people which are specified.

This is a **linear ode**.

- **Logistic Growth**

  Suppose r is the rate of growth on absence of any limiting factor in a population $P(t)$ and k is the environmental capacity factor, where r, k and $P(0)$ are specified. then the growth can be estimated by:

  $$\boxed{\frac{dP}{dt} = P(r - kP)}$$

  That is, $P' = rP - kP^2$ which is a **Bernoulli ode**.

  Let $v = P^{1-2} = P^{-1} \implies v' = -P^{-2}P'$

  Divide the original equation by $-P^2$ to get

$\frac{-P'}{P^2} = \frac{-r}{P} - k$ and substitute to get $v' = -rv + k$

Write the standard form for linear equation and solve : $v' + rv = k \implies \mu = e^{\int r dt} = e^{rt}$

$$v = \frac{1}{\mu}(\int ke^{rt} dt) = e^{-rt}(\frac{ke^{rt}}{r} + C) = \frac{k}{r} + Ce^{-rt}$$

So $P^{-1} = \frac{k}{r} + Ce^{-rt}$

So $P(t) = \frac{1}{\frac{k}{r} + Ce^{-rt}}$ or $\boxed{P(t) = \frac{r}{k + Ce^{-rt}}}$

Then use $P(0)$ to find C.

- **Newton's Law of Cooling**

  Let $T(t)$ denote the temperature of an object and let $M(t)$ be the temperature of the surrounding environment . Newton's law of cooling states that the rate of change of temperature of the object is proportional to the difference between the object and environment temperatures. The differential equation is

$$\boxed{T'(t) = -k(T(t) - M(t))}$$

Here k is a positive constant. Notice the negative sign. If the object is at a higher temperature than the environment, then $T'(t)$ is negative and the temperature decreases and if the object is at a lower temperature than the environment, then $T'(t)$ is positive and the temperature increases. $M(t)$, k, and the initial temperature $T(0)$ must be specified. This is another example of a **linear ode**.