Electrical Circuits

Consider a typical series **RC** circuit with voltage source $v(t)$. Let $i(t)$ denotes the current at time t and $q(t)$ denotes the charge.

Here R is the resistance in ohm (Ω) of the resistor and C is the capacitance in Farad of the capacitor.

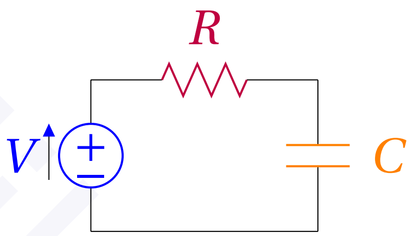
R, C, $v(t)$ and the initial current $i(0)$ are given. The goal is to solve for $i(t)$, the current.

Remember from Physics:

> Voltage drop across the resistor is $R * i(t)$.
>
> Voltage drop across the capacitor is $\frac{1}{C} * q(t)$.

Voltage drop equation is: $R * i(t) + \frac{1}{C} * q(t) = v(t)$

Differentiate both sides to get an ode for $i(t)$:



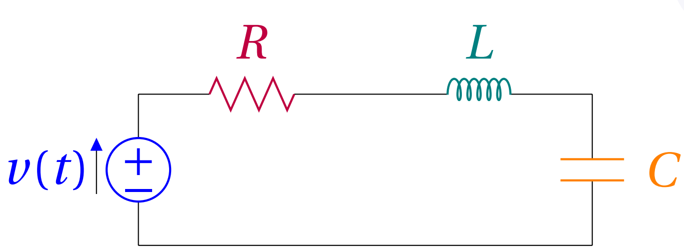
$$R * i'(t) + \frac{1}{C} * i(t) = v'(t)$$

Note that R and C are constants.

This is another example of a **linear ode** and its standard form: $\boxed{i'(t) + \frac{1}{RC} i(t) = \frac{v'(t)}{R}}$

You can add an inductance L in Henry serial in that circuit to get :

$$\boxed{L * i''(t) + R * i'(t) + \frac{1}{C} * i(t) = v'(t)}$$

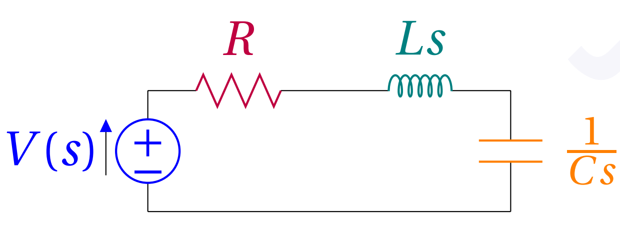


If we start with the circuit at rest, i.e. $i(0) = 0$ and $i'(0) = 0$, one way is take the Laplace transform of the equation to get

$$L * s^2 I(s) + R * sI(s) + \frac{1}{C} * I(s) = sV(s)$$

or $\boxed{Ls * I(s) + R * I(s) + \frac{1}{Cs} * I(s) = V(s)}$

The other method is to think of an inductance as an impedance of Ls, a capacitance as an impedance of $\frac{1}{Cs}$ and a resistance as an impedance R. This is, after performing Laplace transform on the circuit, the following circuit is obtained. Now we can solve the circuit using a a Kirchhoff type loop and impedance replacing resistance at each circuit component:



One benefit of using Laplace transform is that the questions can be set in the Laplace universe and the the inverse can be found.

Example of Impulse force

Solve:

$$y'' + 4y = 2\delta(t - \tfrac{\pi}{4})$$
$$y(0) = 0$$
$$y'(0) = 0$$

Solution:

$$Y(s) = \frac{2e^{-\frac{\pi}{4}s}}{s^2 + 4}$$

$$H(s) = \frac{2}{s^2 + 4} \text{ so } h(t) = \sin(2t)$$

$$y(t) = \sin\Big(2(t - \pi/4)\Big) u_{\pi/4}(t)$$