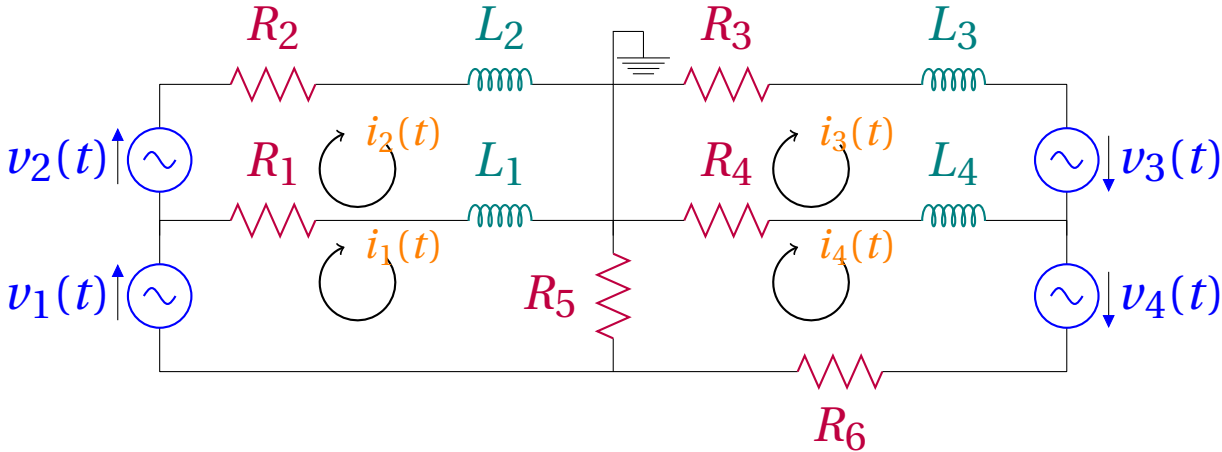# Electrical Networks

Above a primitive simplified electrical network where power plants are placed in different location, they provide power to different areas and they are connected to each other so they supplement each others power. *

The system for this circuit starting from rest is: †

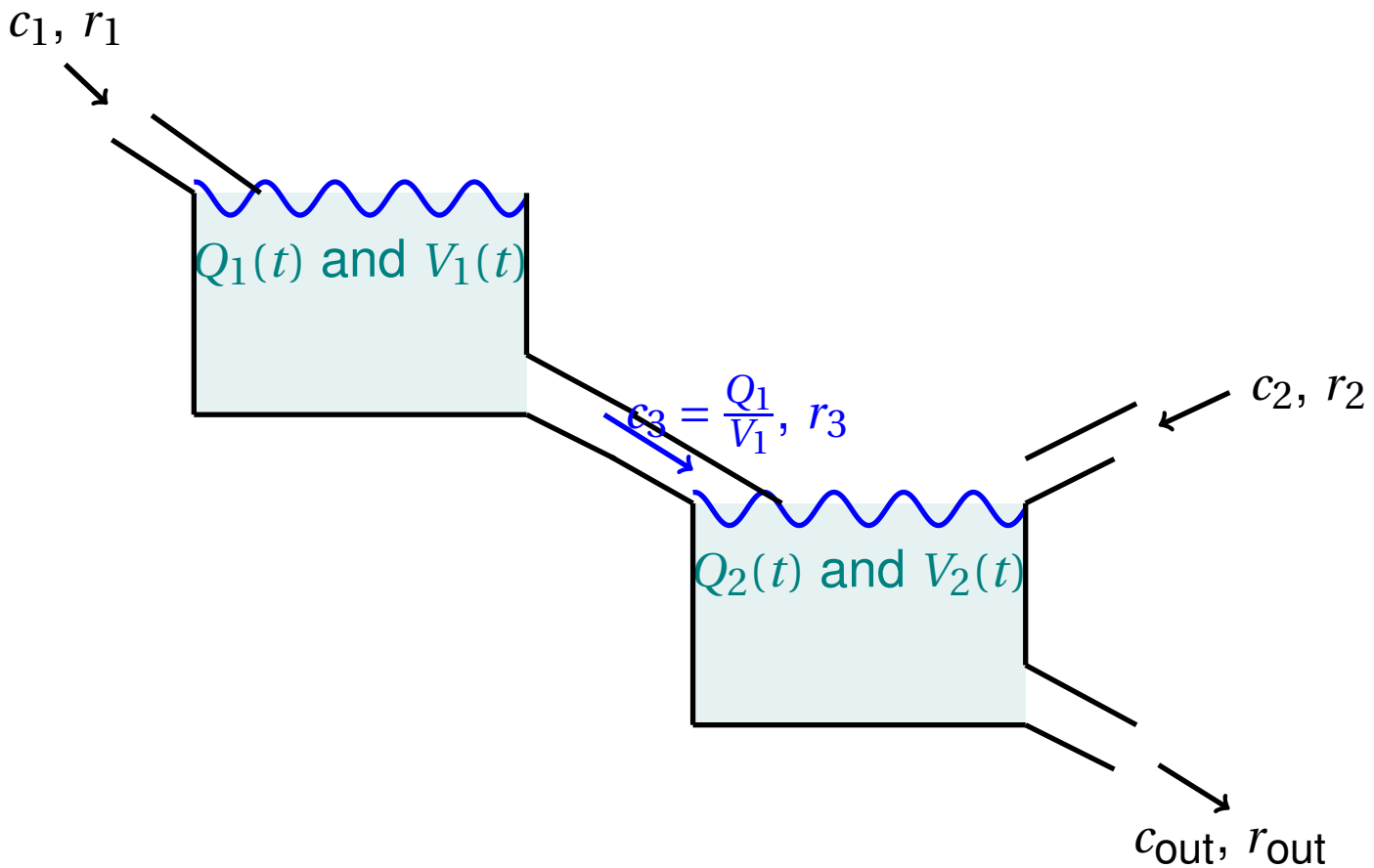*This example is only used to explain the need for systems.
†Note that this is in the Laplace universe.

$$\begin{cases} (R_1 + L_1 s)(I_1 - I_2) + R_5(I_1 - I_4) & = V_1 \\[2mm] (R_1 + L_1 s)(I_2 - I_1) + (R_2 + L_2 s)I_2 & = V_2 \\[2mm] (R_4 + L_4 s)(I_3 - I_4) + (R_3 - L_3 s)I_3 & = V_3 \\[2mm] R_6 I_4 + R_5(I_4 - I_3) + (R_4 + L_4 s)(I_4 - I_3) & = V_4 \end{cases}$$

Here all impedance is known and the voltage that the power plants are producing is known. To find the current, we use the system:

$$\begin{cases} R_1(i_1 - i_2) + L_1(i_1' - i_2') + R_5(i_1 - i_4) = v_1 \\[2mm] R_1(i_2 - i_1) + L_1(i_2' - i_1') + R_2 i_2 + L_2 i_2' = v_2 \\[2mm] R_4(i_3 - i_4) + L_4(i_3' - i_4') + R_3 i_3 + L_3 i_3' = v_3 \\[2mm] R_6 i_4 + R_5(i_4 - i_1) + R_4(i_4 - i_3) + L_4(i_4' - i_3') = v_4 \end{cases}$$

More Complex Tank Problems:

c_1, r_1

$Q_1(t)$ and $V_1(t)$

$c_3 = \frac{Q_1}{V_1}, r_3$

c_2, r_2

$Q_2(t)$ and $V_2(t)$

c_{out}, r_{out}

The problem is more interesting if there are a few tanks and they are connected to each other. But here you can see that the amount in Tank 2 depends on the amount in the other tank.

$$\begin{cases} Q_1' & = r_1 c_1 - r_3 \dfrac{Q_1}{V_1} \\ Q_2' & = r_2 c_2 + r_3 \dfrac{Q_1}{V_1} - r_{\text{out}} \dfrac{Q_2}{V_2} \end{cases}$$

Example: Two species population be Y and B. Let the rate of growth: r_1 and r_2 be the rate of linear growth without any environmental restriction. γ_1, γ_2, σ_1 and σ_2 be the coefficient of how their population interact with each their own and the other species.

Then the system is:

$$\frac{dY}{dt} = r_1 - \gamma_1 Y - \sigma_1 B$$

$$\frac{dB}{dt} = r_2 - \gamma_2 Y - \sigma_2 B$$

In each example we find differential equations that consist of a few dependent variable and their derivatives. The next example explains how to express a higher order differential equation as a system that we can solve in this course.

Example Express the following higher order differential equation into a system:

$$y''' + 2y'' + 5y' + y = e^t \sin t:$$

Let $x_1 = y$, $x_2 = x_1' = y'$, $x_3 = x_2' = y''$ Then:

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ e^t \sin t \end{bmatrix}$$

Process for an equation of degree n:

Step 1 : Rename y and the first $n-1$ derivatives of it as x_1, x_2,...,x_{n-1}

Step 2: The first $n-1$ equations are made from the derivative relationship, e.g. $x_2 = x_1'$, $x_3 = x_2'$.

Step 3: Last equation is found by replacing y by x_1, y' by x_2,... in the original equation.

General form of a Homogeneous System with Constant Coefficients:

$$\vec{X'} = A\vec{X}$$

Where A is an $n \times n$ matrix of coefficients, $\vec{X}$ is $n \times 1$ matrix of variables and $\vec{X'}$ is the $n \times 1$ matrix contains the derivatives of entries of $\vec{X}$.

Example:

$$\begin{cases} x_1' = ax_1 + bx_2 \\ x_2' = cx_1 + dx_2 \end{cases}$$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The General Form of a Linear Non-homogeneous Equation with Constant Coefficients is:

$$\vec{X'} = A\vec{X} + \vec{g}(t)$$